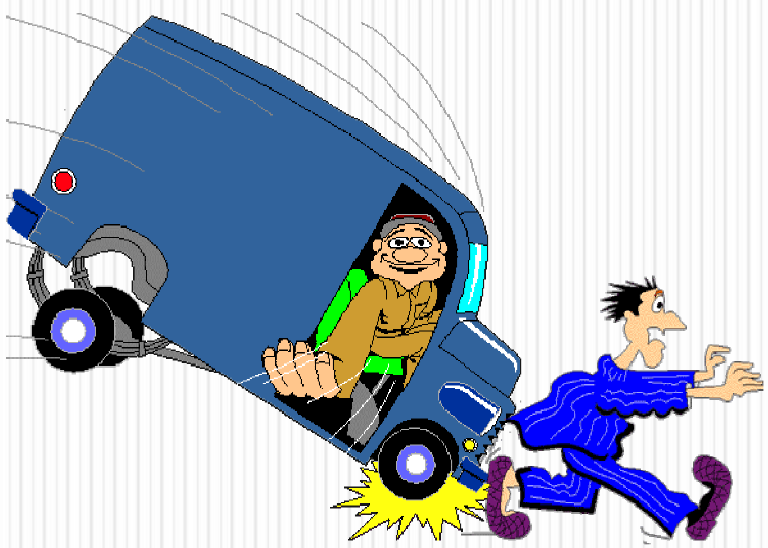
Impulse and Momentum

AP Physics B



Impulse = Momentum

Consider Newton's 2nd Law and the definition of acceleration

Impulse-Momentum Theorem

$J = \Delta p$

$Ft = \Delta mv$

$$\frac{F_{Net}}{m} = a, \quad a = \frac{\Delta v}{t}$$

$$\frac{F_{Net}}{m} = \frac{\Delta v}{t} \rightarrow Ft = \Delta mv$$

$$Ft = \text{Impulse(J)}$$

$$\Delta mv = \text{Momentum(p)}$$

Units of Impulse: **Ns**

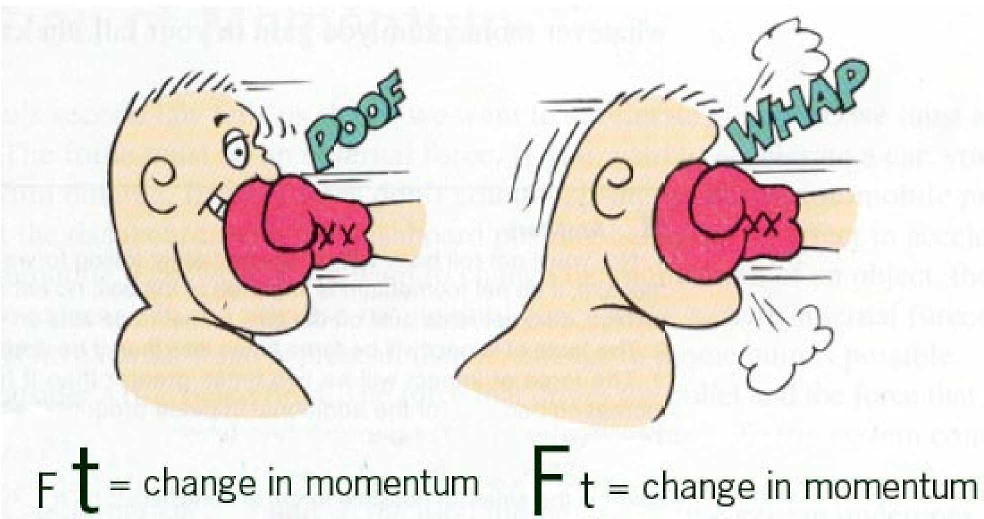
Units of Momentum: **Kg x m/s**

Momentum is defined as ***"Inertia in Motion"***

Impulse – Momentum Theorem

$$Ft = m\Delta v$$

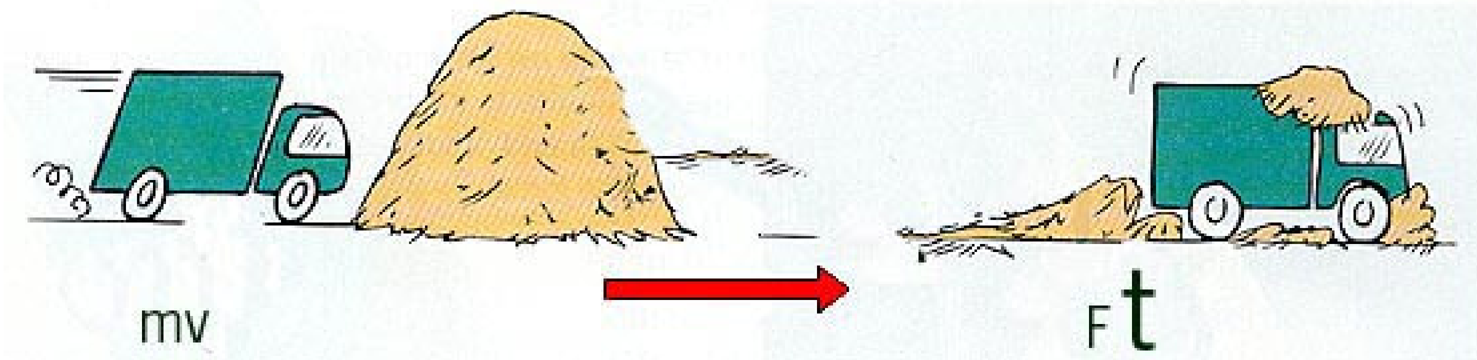
IMPULSE | **CHANGE IN MOMENTUM**

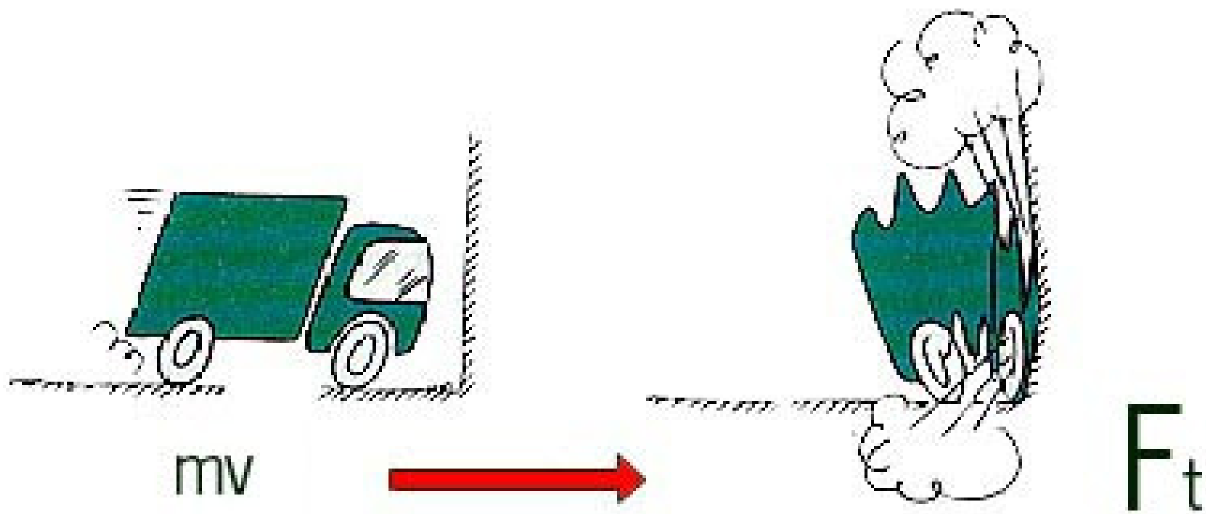


This theorem reveals some interesting relationships such as the INVERSE relationship between FORCE and TIME

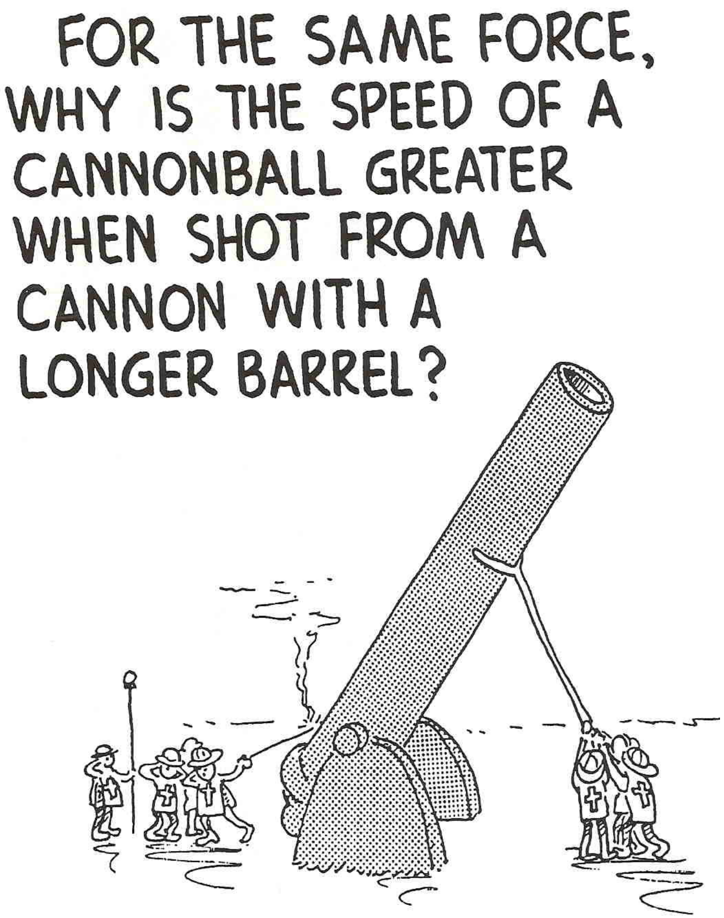
$$F = \frac{m\Delta v}{t}$$

Impulse – Momentum Relationships





Impulse – Momentum Relationships



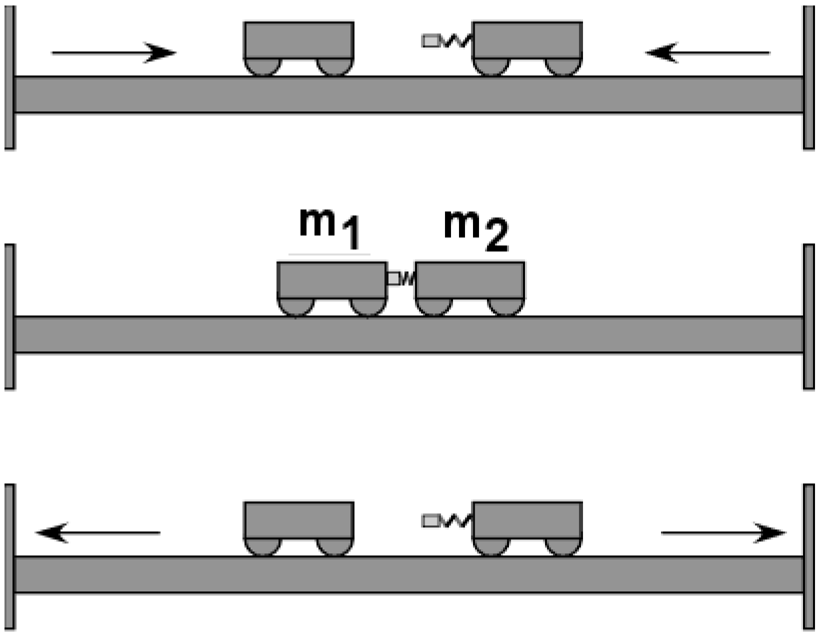
$$fT = m\Delta V$$

Constant

Since TIME is directly related to the VELOCITY when the force and mass are constant, the LONGER the cannonball is in the barrel the greater the velocity.

Also, you could say that the force acts over a larger displacement, thus there is more WORK. The work done on the cannonball turns into kinetic energy.

How about a collision?



Consider 2 objects speeding toward each other. When they collide……

Due to Newton's 3rd Law the FORCE they exert on each other are EQUAL and OPPOSITE.

The TIMES of impact are also equal.

Therefore, the IMPULSES of the 2 objects colliding are also EQUAL

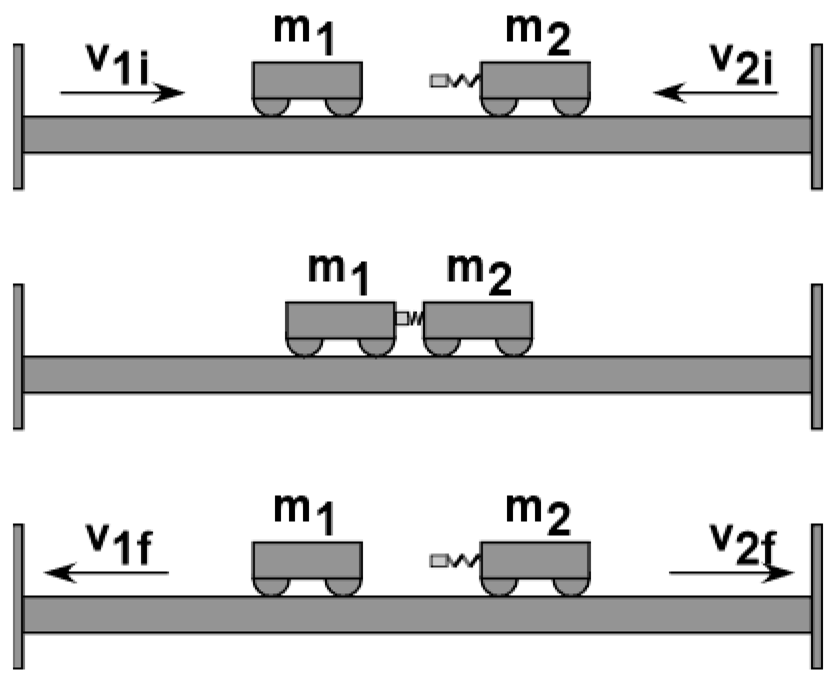
$$F_1 = -F_2 \quad t_1 = t_2$$

$$(Ft)_1 = -(Ft)_2$$

$$J_1 = -J_2$$

How about a collision?

If the Impulses are equal then the MOMENTUMS are also equal!



$$J_1 = -J_2$$

$$p_1 = -p_2$$

$$m_1 \Delta v_1 = -m_2 \Delta v_2$$

$$m_1(v_1 - v_{o1}) = -m_2(v_2 - v_{o2})$$

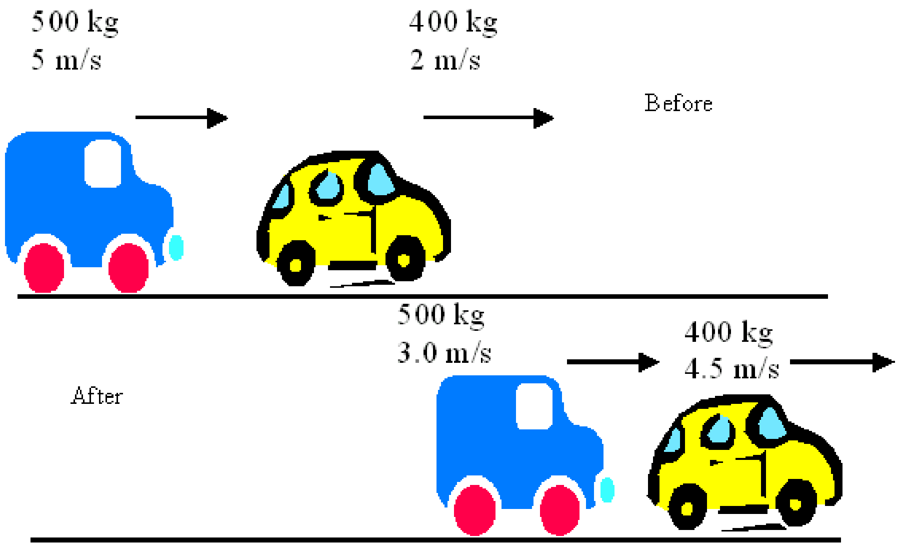
$$m_1 v_1 - m_1 v_{o1} = -m_2 v_2 + m_2 v_{o2}$$

$$\sum p_{before} = \sum p_{after}$$

$$m_1 v_{o1} + m_2 v_{o2} = m_1 v_1 + m_2 v_2$$

Momentum is conserved!

The Law of Conservation of Momentum: ***"In the absence of an external force (gravity, friction), the total momentum before the collision is equal to the total momentum after the collision."***



$$p_{o(truck)} = mv_o = (500)(5) = 2500 kg * m/s$$

$$p_{o(car)} = (400)(2) = 800 kg * m/s$$

$$p_{o(total)} = 3300 kg * m/s$$

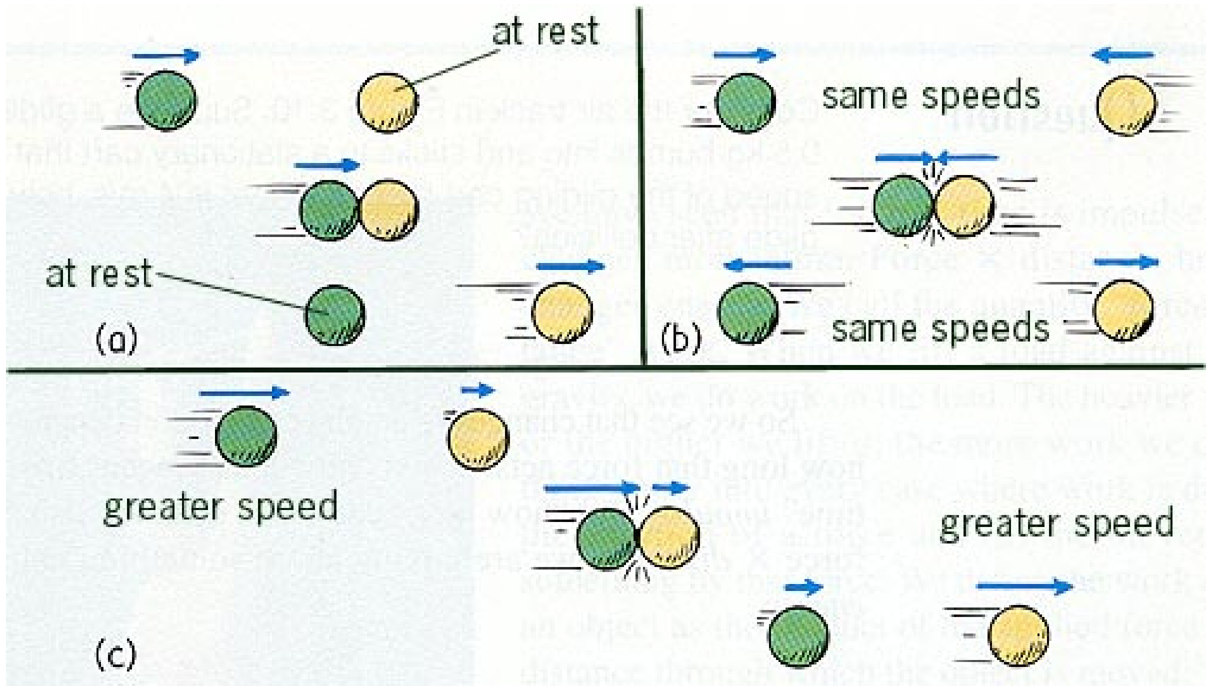
$$p_{truck} = 500 * 3 = 1500 kg * m/s$$

$$p_{car} = 400 * 4.5 = 1800 kg * m/s$$

$$p_{total} = 3300 kg * m/s$$

Types of Collisions

A situation where the objects DO NOT STICK is one type of collision



Notice that in EACH case, you have TWO objects BEFORE and AFTER the collision.

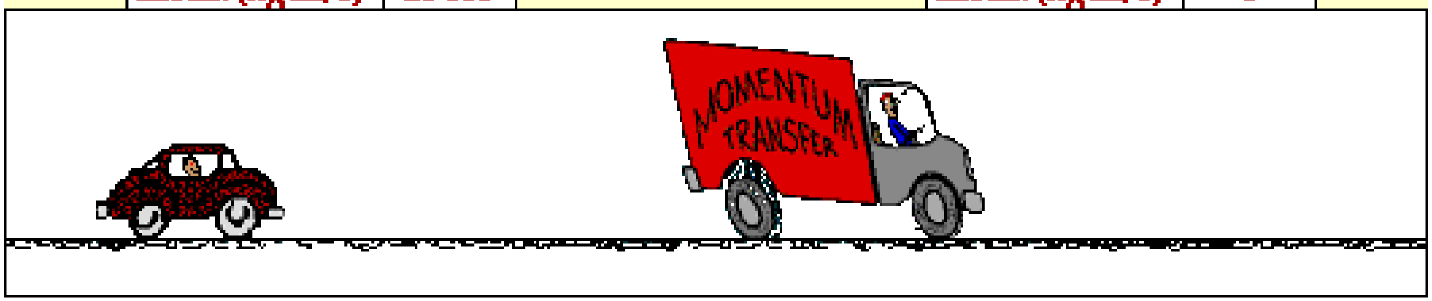
A “no stick” type collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



$\Sigma p_{before} = \Sigma p_{after}$

$$m_1 v_{o1} + m_2 v_{o2} = m_1 v_1 + m_2 v_2$$

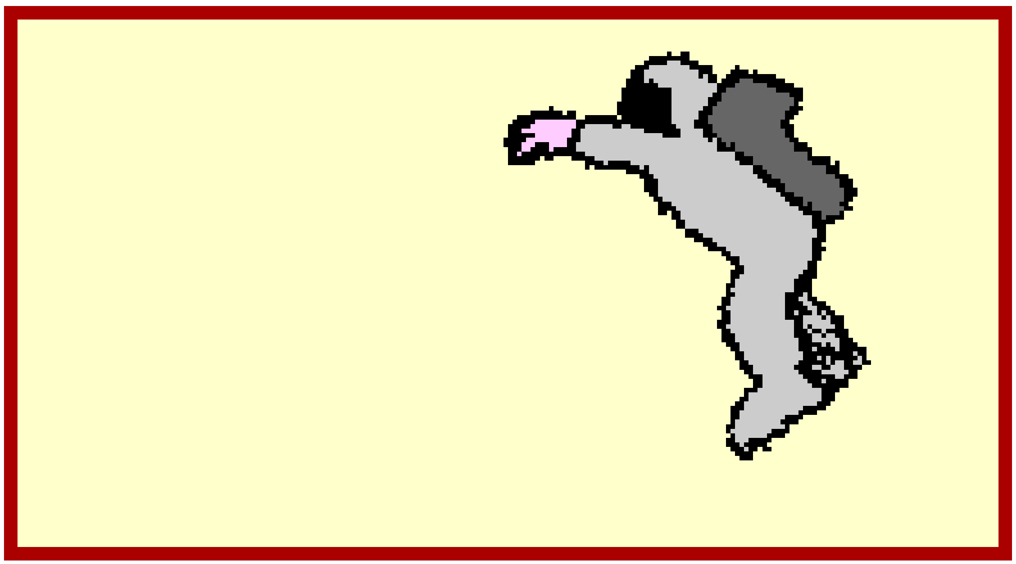
$$(1000)(20) + 0 = (1000)(v_1) + (3000)(10)$$

$$-10000 = 1000 v_1$$

$v_1 =$ **-10 m/s**

Types of Collisions

Another type of collision is one where the objects "STICK" together. Notice you have TWO objects before the collision and ONE object after the collision.



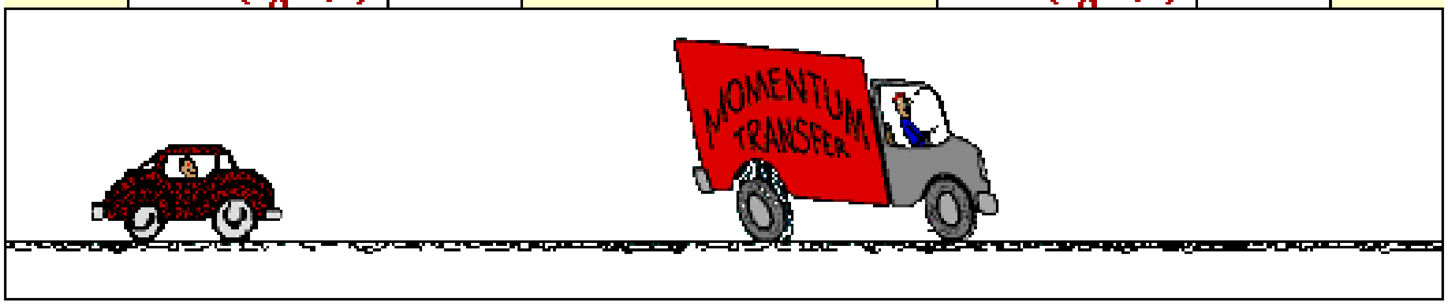
A “stick” type of collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



$$\Sigma p_{before} = \Sigma p_{after}$$

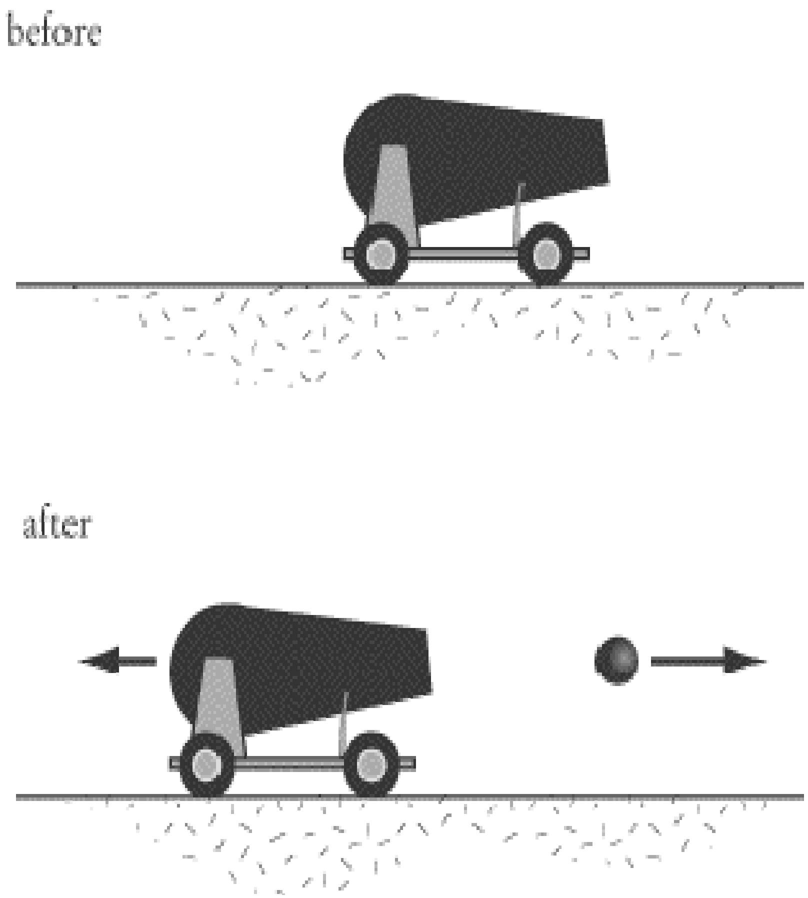
$$m_1 v_{o1} + m_2 v_{o2} = m_T v_T$$

$$(1000)(20) + 0 = (4000) v_T$$

$$20000 = 4000 v_T$$

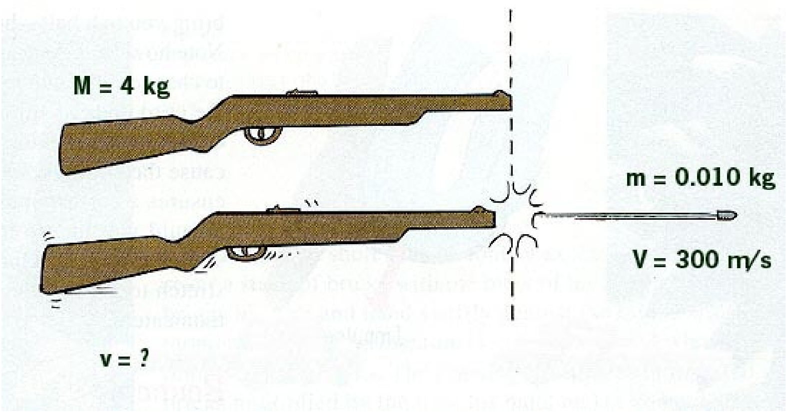
$$v_T =$$ **5 m/s**

The “explosion” type



This type is often referred to as “backwards inelastic”. Notice you have ONE object (we treat this as a SYSTEM) before the explosion and TWO objects after the explosion.

Backwards Inelastic - Explosions



Suppose we have a 4-kg rifle loaded with a 0.010 kg bullet. When the rifle is fired the bullet exits the barrel with a velocity of 300 m/s. How fast does the gun RECOIL backwards?

Σp_{before} =	Σp_{after}
$m_T v_T =$	$m_1 v_1 + m_2 v_2$
$(4.010)(0) =$	$(0.010)(300) + (4)(v_2)$
$0 =$	$3 + 4v_2$
$v_2 =$	**-0.75 m/s**

Collision Summary

Sometimes objects stick together or blow apart. In this case, momentum is ALWAYS conserved.

$$\sum p_{before} = \sum p_{after}$$

$$m_1 v_{01} + m_2 v_{02} = m_1 v_1 + m_2 v_2$$ → When 2 objects collide and DON'T stick

$$m_1 v_{01} + m_2 v_{02} = m_{total} v_{total}$$ → When 2 objects collide and stick together

$$m_{total} v_{o(total)} = m_1 v_1 + m_2 v_2$$ → When 1 object breaks into 2 objects

Elastic Collision = Kinetic Energy is Conserved

Inelastic Collision = Kinetic Energy is NOT Conserved

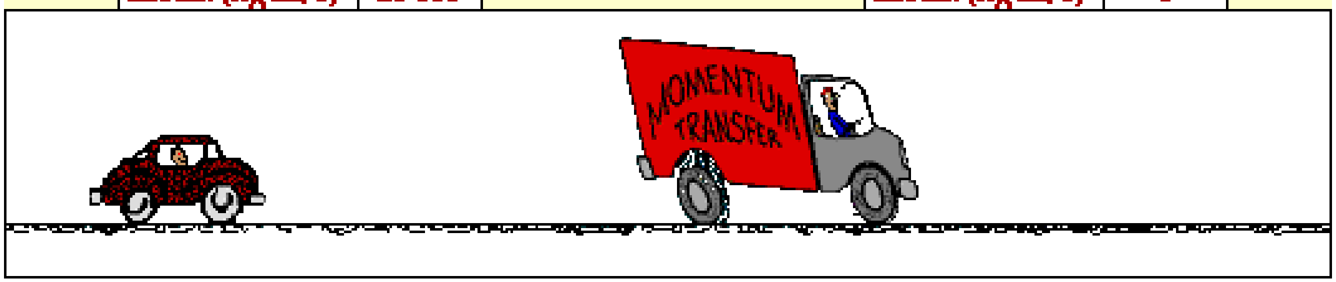
Elastic Collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



$$KE_{car}(Before) = \frac{1}{2}mv^2 = 0.5(1000)(20)^2 = 200{,}000J$$

$$KE_{truck}(After) = 0.5(3000)(10)^2 = 150{,}000J$$

$$KE_{car}(After) = 0.5(1000)(-10)^2 = 50{,}000J$$

Since KINETIC ENERGY is conserved during the collision we call this an **ELASTIC COLLISION**.

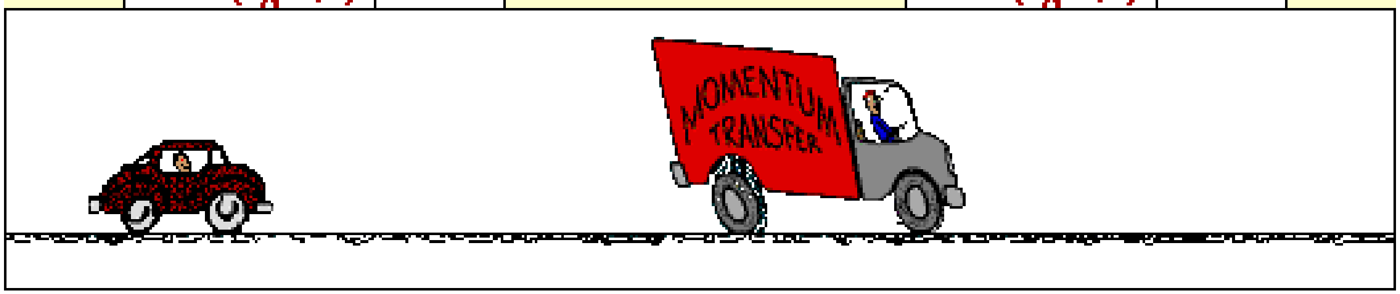
Inelastic Collision

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0

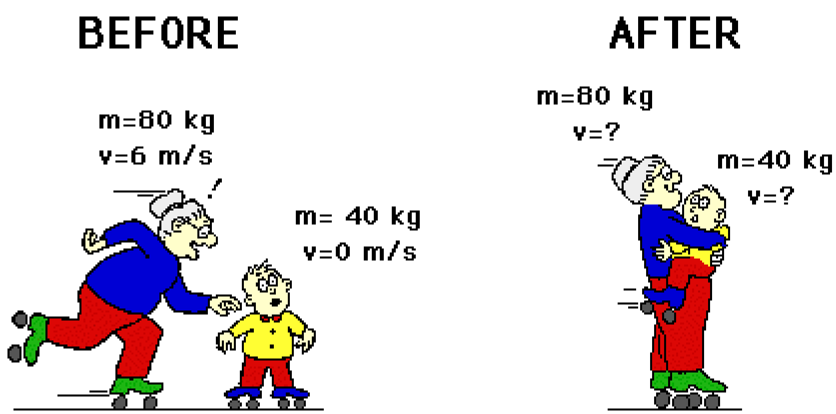


$$KE_{car}(Before) = \frac{1}{2}mv^2 = 0.5(1000)(20)^2 = 200{,}000J$$

$$KE_{truck/car}(After) = 0.5(4000)(5)^2 = 50{,}000J$$

Since KINETIC ENERGY was NOT conserved during the collision we call this an **INELASTIC COLLISION**.

Example



Granny (m=80 kg) whizzes around the rink with a velocity of 6 m/s. She suddenly collides with Ambrose (m=40 kg) who is at rest directly in her path. Rather than knock him over, she picks him up and continues in motion without "braking." Determine the velocity of Granny and Ambrose.

How many objects do I have before the collision?

2

How many objects do I have after the collision?

1

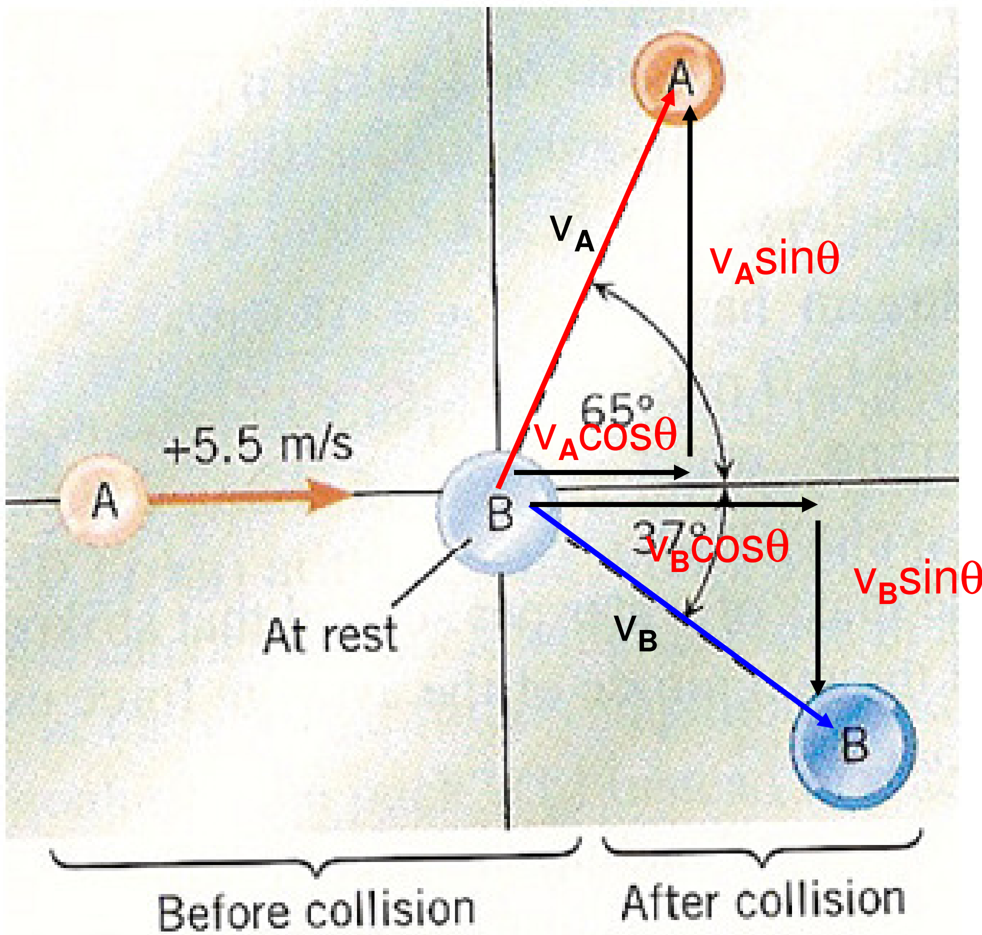
$$\sum p_b = \sum p_a$$

$$m_1 v_{o1} + m_2 v_{o2} = m_T v_T$$

$$(80)(6) + (40)(0) = 120 v_T$$

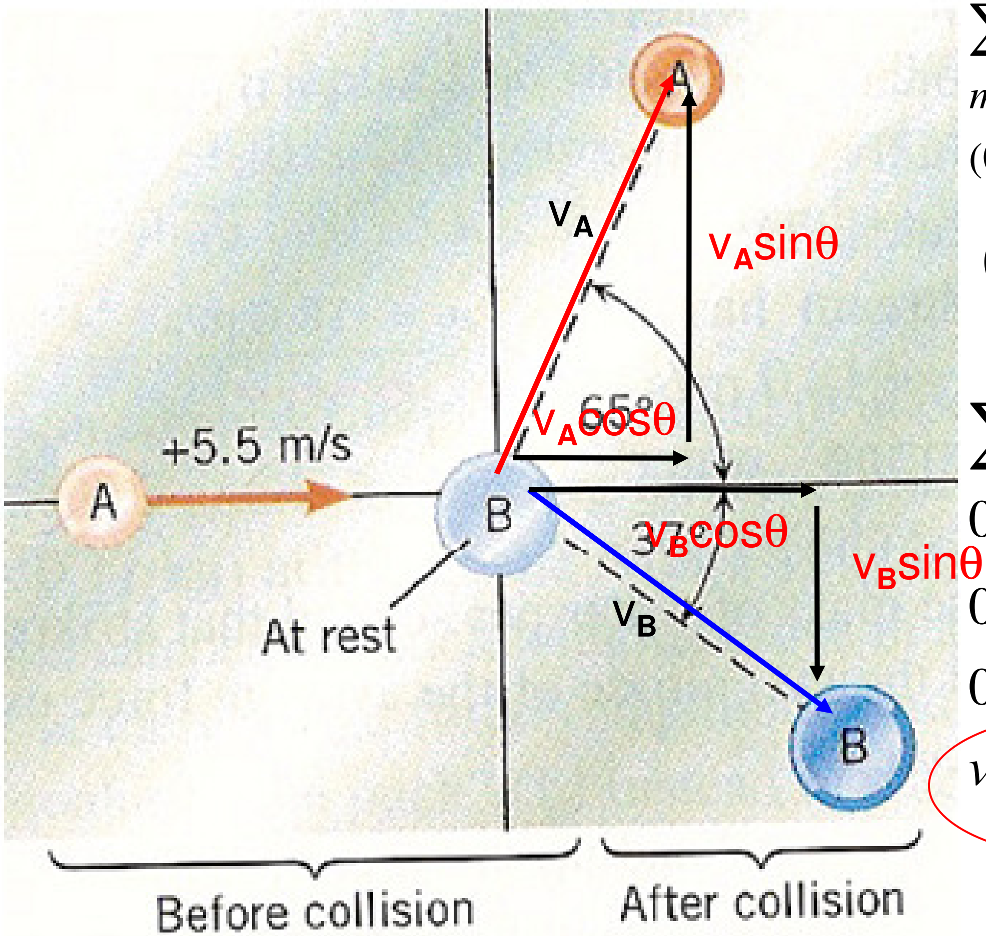
$$v_T = \text{ 4 m/s}$$

Collisions in 2 Dimensions



The figure to the left shows a collision between two pucks on an air hockey table. Puck A has a mass of 0.025-kg and is moving along the x-axis with a velocity of +5.5 m/s. It makes a collision with puck B, which has a mass of 0.050-kg and is initially at rest. The collision is NOT head on. After the collision, the two pucks fly apart with angles shown in the drawing. Calculate the speeds of the pucks after the collision.

Collisions in 2 dimensions



$$\sum p_{ox} = \sum p_x$$

$$m_A v_{oxA} + m_B v_{oxB} = m_A v_{xA} + m_B v_{xB}$$

$$(0.025)(5.5) + 0 = (.025)(v_A \cos 65) + (.050)(v_B \cos 37)$$

$$0.1375 = 0.0106 v_A + 0.040 v_B$$

$$\sum p_{oy} = \sum p_y$$

$$0 = m_A v_{yA} + m_B v_{yB}$$

$$0 = (0.025)(v_A \sin 65) + (0.050)(-v_B \sin 37)$$

$$0.0300 v_B = 0.0227 v_A$$

$$v_B = 0.757 v_A$$

Collisions in 2 dimensions

$$0.1375 = 0.0106v_A + 0.040v_B$$

$$v_B = 0.757v_A$$

$$0.1375 = 0.0106v_A + (0.050)(0.757v_A)$$

$$0.1375 = 0.0106v_A + 0.03785v_A$$

$$0.1375 = 0.04845v_A$$

$$v_A = 2.84m/s$$

$$v_B = 0.757(2.84) = 2.15m/s$$