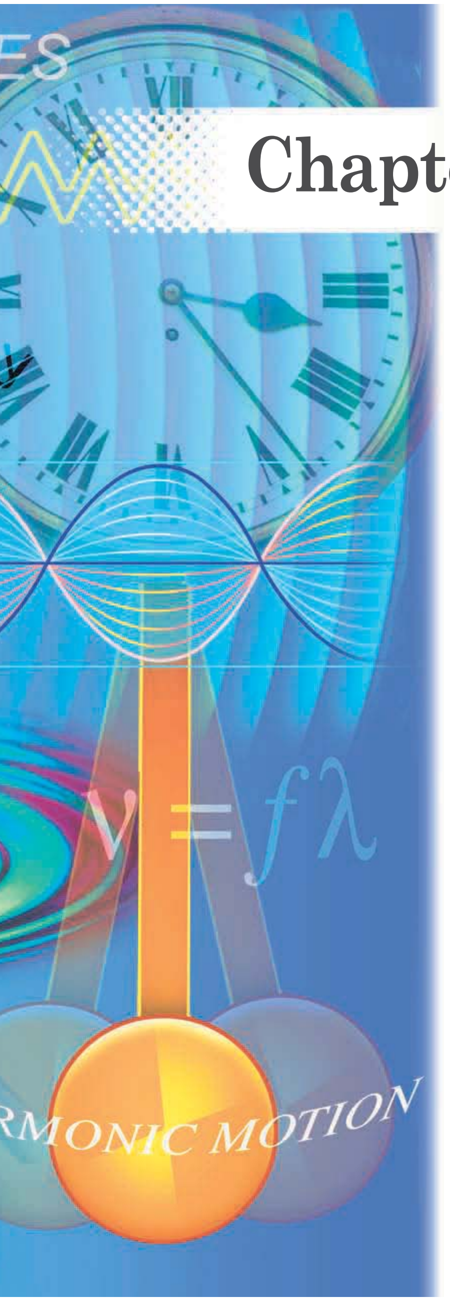

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Chapter 19

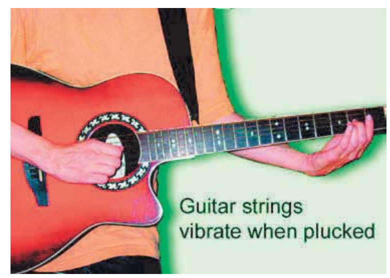
Harmonic Motion

People often create habits that are repetitive because certain repetitive motions have regular comfortable rhythms. Babies like the feel of the back and forth motion of a rocking chair. It seems to make them feel happy and puts them to sleep. We see back and forth motion in many situations. Earth spins you around every 24 hours. Maybe this explains why we are often very comfortable with motions that have regular rhythms.

We see back-and-forth motion in many situations. A swing, the pendulum of a grandfather clock, and a rocking chair all have this kind of motion. Motion that repeats is called harmonic motion. Offered a choice to sit in a regular chair or a rocking chair, you might pick the rocking chair. For one thing, rocking back and forth is more fun than sitting still.

Harmonic motion includes motion that goes around and around. Earth orbiting the sun, the planet spinning on its axis, and a ferris wheel are all examples of this kind of harmonic motion.

Objects or systems that make harmonic motions are called oscillators. Think about where you see oscillators or oscillating systems in your school and home. Look around your classroom — where do you see oscillators? Where do you see back-and-forth motion or motion that goes around and around?


Guitar strings vibrate when plucked

Key Questions

- ✓ How many examples of harmonic motion exist in an amusement park?
- ✓ What do two "out of phase" oscillators look like?
- ✓ How is harmonic motion related to playing a guitar?

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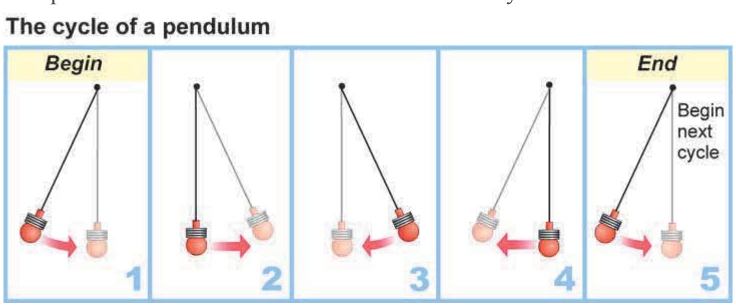
19.1 Harmonic Motion

The forward rush of a cyclist pedaling past you on the street is called *linear motion*. Linear motion gets us from one place to another whether we are walking, riding a bicycle, or driving a car (Figure 19.1). The pedaling action and turning of the cyclist's wheels are examples of harmonic motion. **Harmonic motion** is motion that repeats.

Motion in cycles

What is a cycle? In earlier chapters we used position, speed and acceleration to describe motion. For harmonic motion we need some new ideas that describe the "over-and-over" repetition. The first important idea is the **cycle**. A cycle is a unit of motion that repeats over and over. One spin of a bicycle wheel is a cycle and so is one turn of the pedals. One full back-and-forth swing of a child on a playground swing is also one cycle (Figure 19.1).

Looking at one cycle A pendulum's cycle is shown in the diagram below. Each box in the diagram is a snapshot of the motion at a different time in the cycle.



The cycle of a pendulum The cycle starts with (1) the swing from left to center. Next, the cycle continues with (2) center to right, and (3) back from right to center. The cycle ends when the pendulum moves (4) from center to left because this brings the pendulum back the the beginning of the next cycle. The box numbered "5" is the same as the one numbered "1" and starts the next cycle. Once a cycle is completed, the next cycle begins without any interruption in the motion.

Vocabulary

harmonic motion, cycle, oscillation, oscillator, vibration, period, frequency, hertz, amplitude, damping

Objectives

- ✓ Identify a cycle of harmonic motion.
- ✓ Recognize common oscillators.
- ✓ Know the relationship between period and frequency.
- ✓ Understand how to identify and measure amplitude.

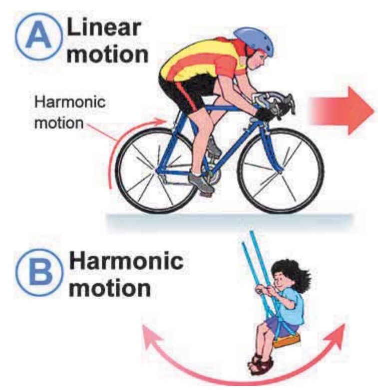


Figure 19.1: *(A) Real-life situations such as riding a bicycle can include both linear motion and harmonic motion. (B) A person on a swing is an example of harmonic motion in action.*

Where do you find harmonic motion?

Oscillators The word **oscillation** means a motion that repeats regularly. Therefore, a system with harmonic motion is called an **oscillator**. A pendulum is an oscillator; so is your heart and its surrounding muscles. Our solar system is a large oscillator with each planet in harmonic motion around the sun. An atom is a small oscillator because its electrons vibrate around the nucleus. The term **vibration** is another word used for back and forth. People tend to use "vibration" for motion that repeats fast and "oscillation" for motion that repeats more slowly.

Earth is part of harmonic motion systems Earth is a part of several oscillating systems. The Earth-sun system has a cycle of one year, which means Earth completes one orbit around the sun in a year. The Earth-moon system has a cycle of approximately 28 days. Earth itself has several different cycles (Figure 19.2). It rotates on its axis once a day, making the 24-hour cycle of day and night. There is also a wobble of Earth's axis that cycles every 22,000 years, moving the north and south poles around by hundreds of miles. There are cycles in weather, such as the El Niño Southern Oscillation, an event that involves warmer ocean water and increased thunderstorm activity in the western Pacific Ocean. Cycles are important; the lives of all plants and animals depend on seasonal cycles.

Music Sound is a traveling vibration of air molecules. Musical instruments and stereo speakers are oscillators that we design to create sounds with certain cycles that we enjoy hearing. When a stereo is playing, the speaker cone moves back and forth rapidly (Figure 19.3). The cyclic back-and-forth motion pushes and pulls on air, creating tiny oscillations in pressure. The pressure oscillations travel to your eardrum and cause it to vibrate. Vibrations of the eardrum move tiny bones in the ear setting up more vibrations that are transmitted by nerves to the brain. There is harmonic motion at every step of the way, from the musical instrument's performance to the perception of sound by your brain.

Color Light is the result of harmonic motion of the electric and magnetic fields (chapter 18). The colors that you see in a picture come from the vibration of electrons in the molecules of paint. Each color of paint contains different molecules that oscillate with different cycles to create the different colors of light you see (chapter 24).

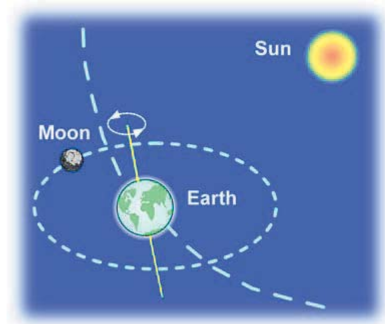


Figure 19.2: *The Earth-sun-moon system has many different cycles. The year, month, and day are the result of orbital cycles.*

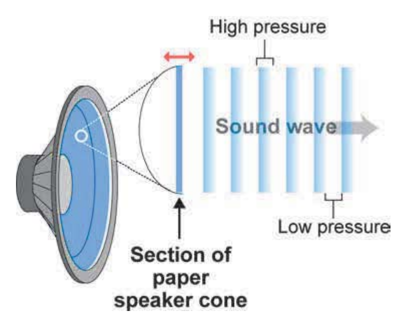


Figure 19.3: *As a speaker cone moves back and forth, it pushes and pulls on air, creating oscillating changes in pressure that we can detect with our ears. The dark blue bands in the graphic represent high pressure regions and the white bands represent low pressure regions.*

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Describing harmonic motion

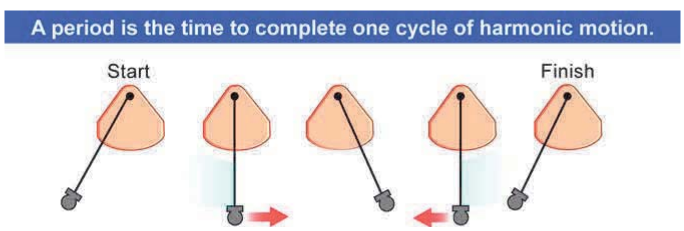
Oscillators in communications

Almost all modern communication technology relies on harmonic motion. The electronic technology in a cell phone uses an oscillator that makes more than 100 million cycles each second (Figure 19.4). When you tune into a station at 101 on the FM dial, you are actually setting the oscillator in your radio to 101,000,000 cycles per second.

Figure 19.4: *The cell phone you use has an electronic oscillator at millions of cycles per second.*

Period is the time for one cycle

The time for one cycle to occur is called the **period**. The cycles of "perfect" oscillators always repeat with the same period. This makes harmonic motion a good way to keep time. For example, a clock pendulum with a period of one second will complete 60 swings (or cycles) in one minute. A clock keeps track of time by counting cycles of an oscillator.



Frequency is the number of cycles per second

The term **frequency** means the number of cycles per second. FM radio (the "FM" stands for frequency modulation) uses frequencies between 95 million and 107 million cycles per second. Your heartbeat has a frequency between one-half and two cycles per second. The musical note "A" has a frequency of 440 cycles per second. The human voice contains frequencies mainly between 100 and 2,000 cycles per second.

A hertz equals one cycle per second

The unit of one cycle per second is called a **hertz**. You hear music when the frequency of the oscillator in your radio exactly matches the frequency of the oscillator in the transmission tower connected to the radio station (Figure 19.5). A radio station dial set to 101 FM receives music broadcast at a frequency of 101,000,000 hertz or 101 megahertz. Your ear can hear frequencies of sound in the range from 20 Hz to between 15,000 and 20,000 Hz. The Hz is a unit that is the same in both the English and metric systems.

Figure 19.5: *You hear music from your car radio when the oscillator in your radio matches the frequency of the oscillator in the transmission tower connected to the radio station.*

Calculating harmonic motion

Frequency is the inverse of period

Frequency and period are inversely related. The period is the time per cycle. The frequency is the number of cycles per time. For example, if the period of a pendulum is 2 seconds, its frequency is 0.5 cycles per second (0.5 Hz).

PERIOD AND FREQUENCY

$$T = \frac{1}{f}$$

Period (seconds) → T; *Frequency* (hertz) → f

$$f = \frac{1}{T}$$

Frequency (hertz) → f; *Period* (seconds) → T

Calculating frequency

The period of an oscillator is 15 minutes. What is the frequency of this oscillator in hertz?

1. Looking for: You are asked for the frequency in hertz.

2. Given: You are given the period in minutes.

3. Relationships: Convert minutes to seconds using the conversion factor 1 minute/60 seconds; Use the formula: $f = {}^{1}/_{T}$;

4. Solution:

$$15\,\text{min} \times \frac{60\,\text{sec}}{1\,\text{min}} = 900\,\text{sec};\; f = \frac{1}{900\,\text{sec}} = 0.0011\,\text{Hz}$$

Your turn...

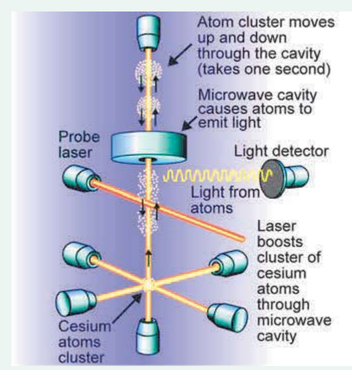
a. The period of an oscillator is 2 minutes. What is the frequency of this oscillator in hertz? **Answer:** 0.008 Hz

b. How often would you push someone on a swing to create a frequency of 0.20 hertz? **Answer:** every 5 seconds

c. The minute hand of a clock pendulum moves 1/60 of a turn after 30 cycles. What is the period and frequency of this pendulum? **Answer:** 60 seconds divided by 30 cycles = 2 seconds per cycle; the period is 2 seconds and the frequency is 0.5 Hz.

Keeping "perfect" time

The world's most accurate clock, the NIST-F1 Cesium Fountain Atomic Clock in Boulder, Colorado, keeps time by counting cycles of light waves emitted by a cluster of cesium atoms. This clock can run for more than 30 million years and not gain or lose a single second! The cesium atoms are cooled to near absolute zero by floating them in a vacuum on a cushion of laser light. The very low temperature is what makes the clock so stable and accurate. At normal temperatures the frequency of light waves would be affected by the thermal motion of the cesium atoms. Near absolute zero the thermal motion is all but eliminated. .



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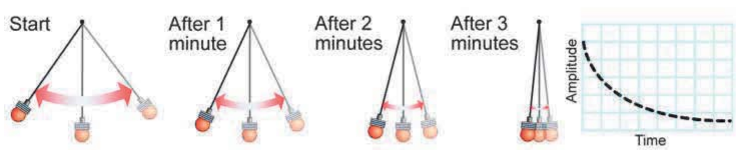
Amplitude

Amplitude describes the size of a cycle

You know the period is the time to complete a cycle. The **amplitude** describes the "size" of a cycle. Figure 19.6 shows a pendulum with small amplitude and large amplitude. With mechanical systems (such as a pendulum), the amplitude is often a distance or angle. With other kinds of oscillators, the amplitude might be voltage or pressure. The amplitude is measured in units appropriate to the kind of system you are describing.

How do you measure amplitude?

The amplitude is the maximum distance the oscillator moves away from its *equilibrium* position. For a pendulum, the equilibrium position is hanging straight down in the center. For the pendulum in Figure 19.7, the amplitude is 20 degrees, because the pendulum moves 20 degrees away from center in either direction.



Damping

Friction slows a pendulum down, as it does all oscillators. That means the amplitude slowly gets reduced until the pendulum is hanging straight down, motionless. We use the word **damping** to describe the gradual loss of amplitude of an oscillator. If you wanted to make a clock with a pendulum, you would have to find a way to keep adding energy to counteract the damping of friction.

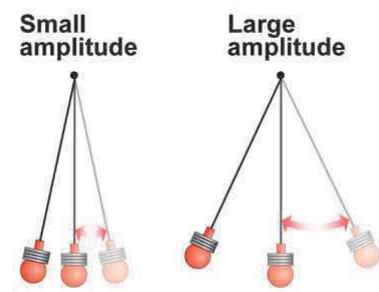


Figure 19.6: *Small amplitude versus large amplitude.*

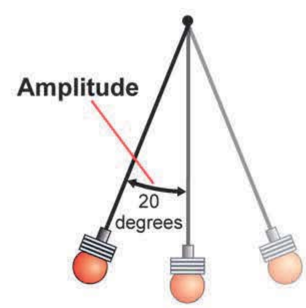


Figure 19.7: *A pendulum with an amplitude of 20 degrees swings 20 degrees away from the center.*

19.1 Section Review

1. Which is the best example of a cycle: a turn of a bicycle wheel or a slide down a ski slope?
2. Describe one example of an oscillating system you would find at an amusement park.
3. What is the relationship between period and frequency?
4. Every 6 seconds a pendulum completes one cycle. What are the period and frequency of this pendulum?

19.2 Graphs of Harmonic Motion

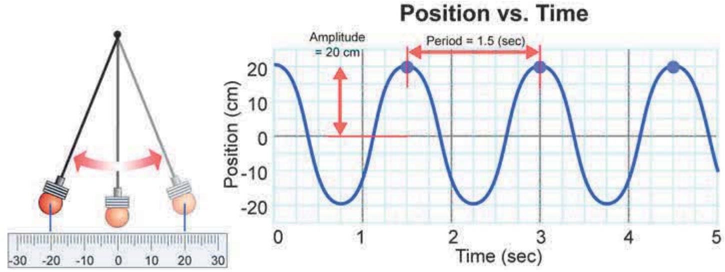
Harmonic motion graphs show cycles (Figure 19.8). Even without seeing the actual motion, you can look at a harmonic motion graph and figure out the period and amplitude. You can also quickly sketch an accurate harmonic motion graph if you know the period and amplitude.

Reading harmonic motion graphs

Repeating patterns The most common type of graph puts position on the vertical (*y*) axis and time on the horizontal (*x*) axis. The graph below shows how the position of a pendulum changes over time. The repeating "wave" on the graph represents the repeating cycles of motion of the pendulum.

Finding the period This pendulum has a period of 1.5 seconds so the pattern on the graph repeats every 1.5 seconds. If you were to cut out any piece of the graph and slide it over 1.5 seconds it would line up exactly. You can tell the period is 1.5 seconds because the graph repeats itself every 1.5 seconds.

Showing amplitude on a graph The amplitude of harmonic motion can also be seen on a graph. The graph below shows that the pendulum swings from +20 centimeters to -20 centimeters and back. Therefore, the amplitude of the pendulum is 20 centimeters. Harmonic motion graphs often use positive and negative values to represent motion on either side of a center (equilibrium) position. Zero usually represents the equilibrium point. Notice that zero is placed halfway up the *y*-axis so there is room for both positive and negative values. This graph is in centimeters but the motion of the pendulum could also have been graphed using the angle measured relative to the center (straight down) position.



Vocabulary

phase

Objectives

- ✓ Recognize the difference between linear motion and harmonic motion graphs.
- ✓ Interpret graphs of harmonic motion.
- ✓ Determine amplitude and period from a harmonic motion graph.
- ✓ Recognize when two oscillators are in phase or out of phase.

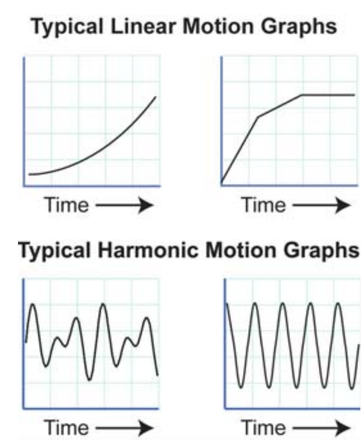


Figure 19.8: *Typical graphs for linear motion (top) and harmonic motion (bottom). Graphs of linear motion do not show cycles. Harmonic motion graphs show repeating cycles.*

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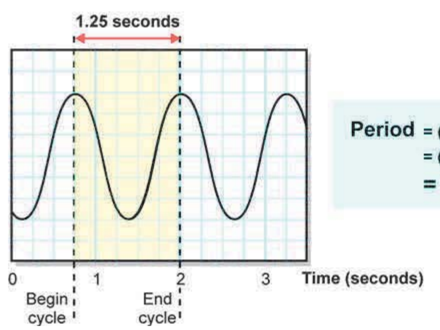
Determining period and amplitude from a graph

Calculating period from a graph

To find the period from a graph, start by identifying one complete cycle. The cycle must begin and end in the same place in the pattern. Figure 19.9 shows how to choose the cycle for a simple harmonic motion graph and for a more complex one. Once you have identified a cycle, you use the time axis of the graph to determine the period. The period is the time difference between the beginning of the cycle and the end. Subtract the beginning time from the ending time, as shown in the example below.



Period = (ending time - beginning time)
= (2.0 sec - 0.75 sec)
= **1.25 seconds**

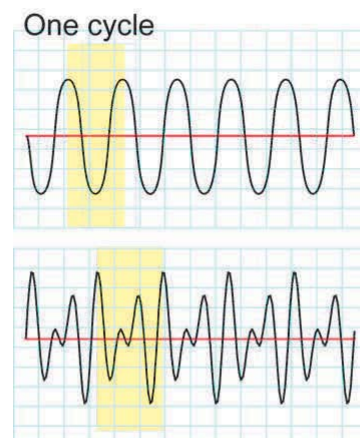


Figure 19.9: *The cycle is the part of the graph that repeats over and over. The yellow shading shows one cycle for each of the graphs above.*

Calculating amplitude from a graph

On a graph of harmonic motion, the amplitude is half the distance between the highest and lowest points on the graph. For example, in Figure 19.10, the amplitude is 20 centimeters. Here is the calculation:

[20 cm - (- 20 cm)] ÷ 2 = [20 cm + 20 cm] ÷ 2 = 40 cm ÷ 2 = 20 cm.

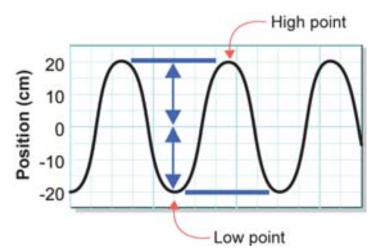


Figure 19.10: *The amplitude is one-half the distance between the highest and lowest points on the graph. In this graph of harmonic motion, the amplitude is 20 centimeters.*

AMPLITUDE

$$Amplitude = \frac{1}{2}\left(\text{high point} - \text{low point}\right)$$

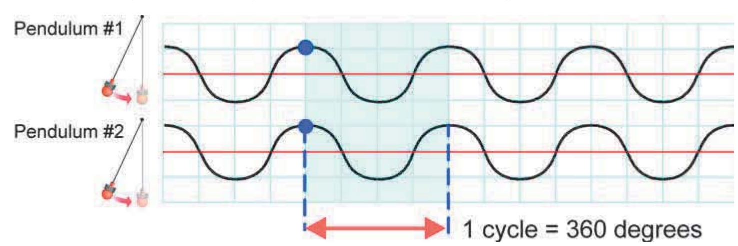
Circular motion and phase

Phase How do you describe where a pendulum is in its cycle? Saying the pendulum is at a 10 degree angle is not enough. If the pendulum started at 10 degrees, then it would be at the start of its cycle. If the pendulum started at 20 degrees it would be part way through its cycle and could be near the start or the end. The **phase** tells you exactly where an oscillator is in its cycle. Phase is measured relative to the whole cycle, and is independent of amplitude or period.

Cycles of circular motion are 360° The most convenient way to describe phase is to think in terms of angles and circular motion. Circular motion is a kind of harmonic motion because rotation is a pattern of repeating cycles. The cycles of circular motion always measure 360 degrees. It does not matter how big the wheel is, each full turn is 360 degrees. Because circular motion always has cycles of 360 degrees, *we use degrees to measure phase*.

Phase is measured in degrees To see how degrees apply to harmonic motion that is not circular (such as a pendulum), imagine a peg on a rotating turntable (Figure 19.11). A bright light casts a shadow of the peg on the wall. As the turntable rotates, the shadow goes back and forth on the wall (A and B in Figure 19.11). If we make a graph of the position of the shadow, we get a harmonic motion graph (C). One cycle passes every 360 degree turn of the turntable. A quarter cycle has a phase of 90 degrees, half a cycle has a phase of 180 degrees and so on (Figure 19.11).

Two oscillators "in phase" The concept of phase is most important when comparing two or more oscillators. Imagine two identical pendulums. If you start them together, their graphs look like the picture below. We say these pendulums are *in phase* because their cycles are aligned. Each is at the same phase at the same time.



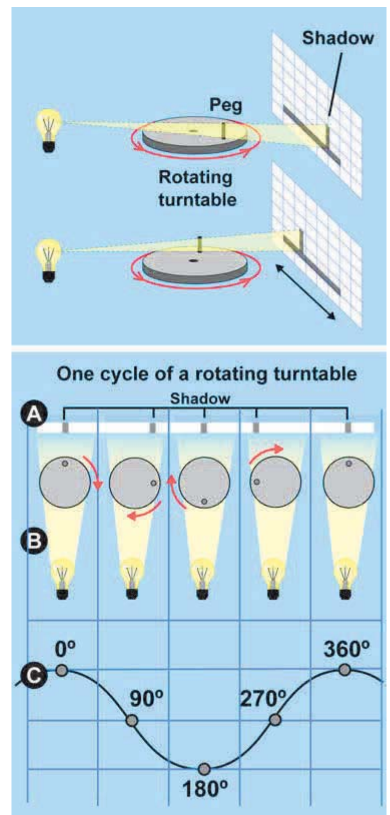


Figure 19.11: *The harmonic motion of a rotating turntable is illustrated by the back-and-forth motion of the shadow of the peg.*

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Harmonic motion that is out of phase

Out of phase by 90 degrees

If we start the first pendulum swinging a little before the second one, the graphs look like Figure 19.12. Although, they have the same cycle, the first pendulum is always a little bit ahead of the second. Notice that the graph for pendulum number 1 reaches its maximum 90 degrees *before* the graph for pendulum number 2. We say the pendulums are *out of phase* by 90 degrees, or one-fourth of a cycle (90 degrees is one-fourth of 360 degrees).

Out of phase by 180 degrees

When they are out of phase, the relative motion of oscillators may differ by a little or by as much as half a cycle. Two oscillators that are 180 degrees out of phase are one-half cycle apart. Figure 19.13 shows that the two pendulums are always on opposite sides of the cycle from each other. When pendulum number 1 is all the way to the left, pendulum number 2 is all the way to the right. This motion is illustrated on the graph by showing that "peaks" of motion (positive amplitude) for one pendulum match the "valleys" of motion (negative amplitude) for the other.

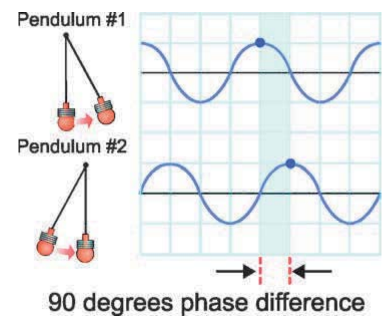


Figure 19.12: *The two pendulums are 90 degrees out of phase.*

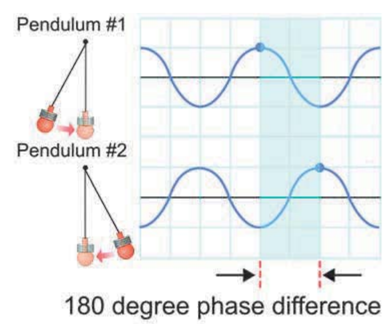


Figure 19.13: *The two pendulums are 180 degrees out of phase.*

19.2 Section Review

1. What is the difference between a graph of linear motion and a graph of harmonic motion?
2. A graph of the motion of a pendulum shows that it swings from +5 centimeters to -5 centimeters for each cycle. What is the amplitude of the pendulum?
3. A pendulum swings from -10 degrees to +10 degrees. What is the amplitude of this pendulum?
4. A graph of harmonic motion shows that one cycle lasted from 4.3 seconds to 6.8 seconds. What is the period of this harmonic motion?
5. A graph of harmonic motion shows that the motion lasted for 10 seconds and it included 5 cycles. What is the period of this harmonic motion?
6. Sketch the periodic motion for two oscillators that are 45 degrees out of phase.
7. If one oscillator were out of phase with another oscillator by 45 degrees, what fraction of a 360-degree cycle would it be out of phase? 1/8, 1/4, 1/2, or 3/4?

19.3 Properties of Oscillators

Why does a pendulum oscillate? A car on a ramp just rolls down and does not oscillate. What properties of a system determine whether its motion will be linear motion or harmonic motion? You will learn the answers to these questions in this section. You will also learn how to change the period of an oscillator by changing the ratio of a few important variables.

Vocabulary

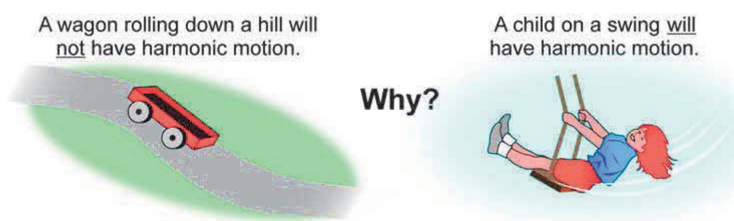
equilibrium, restoring force, natural frequency, periodic force, resonance

Objectives

- ✓ Understand the role of restoring force in how oscillators work.
- ✓ Learn the relationship between amplitude and period for a pendulum.
- ✓ Recognize simple oscillators.

Restoring force and equilibrium

Different kinds of systems If you set a wagon on a hill and let it go, the wagon rolls down and does not come back. If you push a child on a swing, the child goes away from you at first, but then comes back. The child on the swing shows harmonic motion while the wagon on the hill does not. What is the fundamental difference between the two situations?



Equilibrium Systems that have harmonic motion always move back and forth around a central or **equilibrium** position. You can think of equilibrium as the system at rest, undisturbed, with zero net force. A wagon on a hill is *not* in equilibrium because the force of gravity is not balanced by another force. A child sitting motionless on a swing *is* in equilibrium because the force of gravity is balanced by the tension in the ropes.

Restoring forces Equilibrium is maintained by restoring forces. A **restoring force** is any force that always acts to pull the system back toward equilibrium. If the child on the swing is moved forward, gravity creates a restoring force that pulls her back, toward equilibrium. If she moves backward, gravity pulls her forward, back to equilibrium again (Figure 19.14). Systems with restoring forces are the ones that move in harmonic motion.

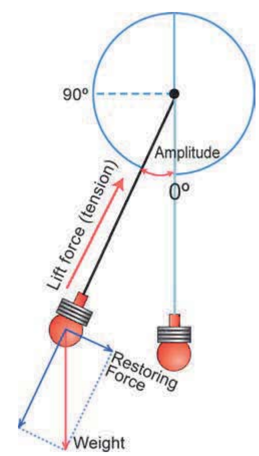


Figure 19.14: *Restoring force keeps a pendulum swinging. Restoring force is related to weight and the lift force (or tension) of the string of a pendulum.*

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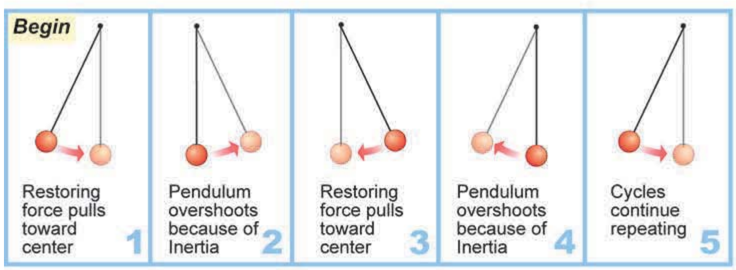
Inertia and mass

Inertia causes an oscillator to go past equilibrium

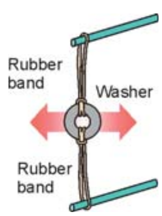
The restoring force of gravity always pulls a pendulum towards equilibrium. Why doesn't the pendulum just stop at equilibrium? Newton's first law of motion explains why. According to the first law, an object in motion tends to stay in motion. The pendulum has inertia that keeps it moving forward. Inertia causes the pendulum to overshoot its equilibrium position every time. The result is harmonic motion.



Inertia is common to all oscillators

All systems that oscillate on their own (without a motor) have some property that acts like inertia and some type of restoring force. Harmonic motion results from the interaction of the two effects: inertia and restoring force.

Increasing mass may increase the period



You can make a simple oscillator with a steel washer and two rubber bands (picture). What happens to the period if you increase the mass by adding more washers? The restoring force from the rubber band is the same. If the mass increases, then (by Newton's second law) the acceleration decreases proportionally. That means the oscillator moves slower and the period gets longer.

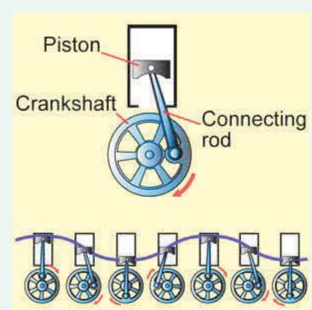
How mass affects the period

Changing the mass of a pendulum does *not* affect its period. That is because the restoring force on a pendulum is created by gravity. Like free fall, if you add mass to a pendulum the added inertia is exactly equal to the added force from gravity. The acceleration is the same and therefore the period stays the same.

Harmonic motion in machines

Natural harmonic motion results from restoring forces and inertia. However, harmonic motion can also be forced. When a machine is involved, cycles of motion can be created using an energy source to push or rotate parts. Mechanical systems usually do not depend on a restoring force or inertia to keep going.

For example, the piston of a car engine goes up and down as the crank turns. The piston is in harmonic motion, but the motion is caused by the rotation of the crankshaft and the attachment of the connecting rod. Gasoline provides the energy to keep this harmonic motion system going.



Period and natural frequency

Natural frequency A pendulum will have the same period each time you set it in motion. Unless you change the pendulum itself (such as changing its length) it will always swing with the same period. The **natural frequency** is the frequency (or period) at which a system naturally oscillates. Every system that oscillates has a natural frequency.

Why natural frequency is important Microwave ovens, musical instruments, and cell phones are common devices that use the natural frequency of an oscillator. For example, the strings of a guitar are tuned by adjusting the natural frequency of vibrating strings to match musical notes (Figure 19.15). All objects can oscillate, and that means everything in the universe has a natural frequency. In fact, most things have several natural frequencies because they can oscillate in different ways.

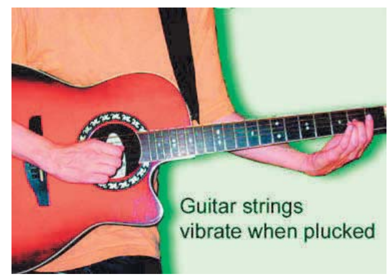
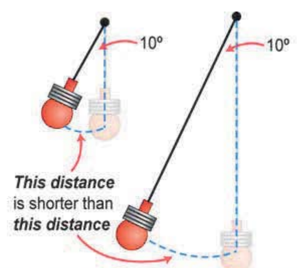


Figure 19.15: *A guitar uses the natural frequency of strings to make musical notes. Here, the musician plays the musical note A. As a result, the string vibrates at 440 hertz.*

Natural frequency



Changing the length of the string changes the period of a pendulum

The natural frequency depends on the balance between restoring force and inertia (mass). Any change that affects this balance will also change the natural frequency. The natural frequency of a pendulum depends on the length of the string. If you make the string longer, the restoring force is spread out over a proportionally greater distance. The period of the pendulum gets longer. Tuning a guitar changes the natural frequency of a string by changing its tightness (or *tension*). Changing the mass changes the natural frequency *only if restoring force is not due to gravity.*

Periodic force You can keep a swing (pendulum) swinging for a long time by pushing it at the right time every cycle. A force that is repeated over and over is called a **periodic force**. A periodic force has a cycle with an amplitude, frequency and period, just like an oscillator. *To supply energy to an oscillator you need to use a periodic force.* A constant force will not have the same effect. If you push once per cycle (periodic force) the amplitude of a swing increases (Figure 19.16). If you applied a constant force of the same strength, the swing would move in the direction of your force and stay there, motionless.

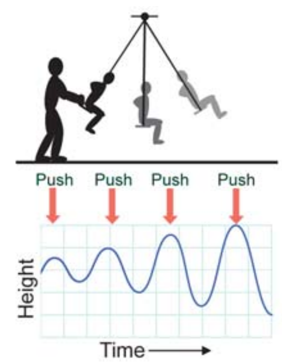


Figure 19.16: *Each push of a swing at the right time increases the amplitude (height) of the swing. Each push is a periodic force.*

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Resonance

Force and natural frequency

Newton's second law ($a = F/m$) tells you how much acceleration you get for a given force and mass. While the second law is still true for harmonic motion, there is a new and important difference. Harmonic motion is motion that oscillates back and forth. What happens if the force is periodic and oscillates back and forth too? When you shake one end of a rope up and down in a steady rhythm you are applying a periodic force to the rope (Figure 19.17). The rope behaves very differently depending on the frequency at which you shake it up and down! If you shake it at *just the right frequency* the rope swings up and down in harmonic motion with a large amplitude. If you don't shake at the right frequency, the rope wiggles around but you don't get the large amplitude *no matter how strong a force you apply.*

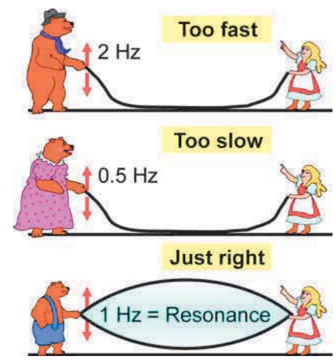

Figure 19.17: *A jump rope is a good example of resonance. If you shake it at the right frequency, it makes a big wave motion. If your frequency is too fast or too slow, the rope will not make the wave pattern at all.*

Resonance

Resonance occurs when a periodic force has the same frequency as the natural frequency of the system. If the force and the motion have the same frequency, each cycle of the force matches a cycle of the motion. As a result each push adds to the next one and the amplitude of the motion grows. You can think about resonance in three steps: the *periodic force*, the *system*, and the *response*. The response is what the system does when you apply the periodic force (Figure 19.18). In resonance, the response is very large compared to the strength of the force, much larger than you would expect. Resonance occurs when:

- there is a system in harmonic motion, like a swing;
- there is a periodic force, like a push;
- the frequency of the periodic force matches the natural frequency of the system.

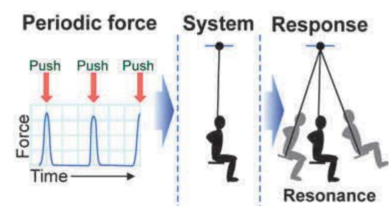

Figure 19.18: *When you push someone on a swing, you are using resonance. Small or big pushes at the right time in a swing's motion will make a person swing higher. If the periodic force (the push) is applied at the wrong time, the swing does not swing at all.*

A jump rope is a example of resonance

Like a swing, a jump rope depends on resonance. If you want to get a jump rope going, you shake the ends up and down. By shaking the ends, you are applying a periodic force to the rope. However, if you have tried to get a jump rope going, you have noticed that you have to get the right rhythm to get the rope moving with a large amplitude (Figure 19.17). The extra-strong response at 1 hertz is an example of resonance and happens only when the frequency (rhythm) of your periodic force matches the natural frequency of the jump rope.

Simple oscillators

A mass on a spring

You know from experience that springs resist being extended or compressed. Figure 19.19 shows how the restoring force from a spring always acts to return it to equilibrium. A system of a mass on a spring is a simple oscillator. When the spring is compressed, it pushes back on the mass. When the spring is extended, it pulls on the mass. The system is an oscillator because the push-pull of the spring is a restoring force and the mass supplies the inertia. An example of a mass on a spring is a car (mass) and its shock absorbers (springs). Wheels on springs can oscillate up and down over bumps without the whole car having to move up and down too. Along with springs, shock absorbers also have high friction *dampers* that quickly slow any oscillation down. A car that keeps bouncing after going over a bump has shock absorbers with dampers that are worn out and not providing enough friction.

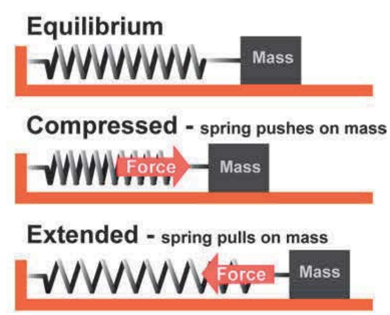

Figure 19.19: *A mass on a spring is an oscillating system. When the spring is compressed, it pushes back on the mass to return to equilibrium. When the spring is extended, it pulls the mass back toward equilibrium.*

A vibrating string

An example of a *vibrating string* oscillator is a rubber band stretched between two rods (Figure 19.20). If the middle of the rubber band is pulled to the side, it will move back toward being straight when it is released. Stretching the rubber band to the side creates a restoring force. When the rubber band is released, inertia carries it past being straight and it vibrates. Vibrating strings tend to move much faster than springs and pendulums. The period of a vibrating string can be one-hundredth of a second (0.01 second) or shorter.

Mass on a vibrating string

You can modify the rubber band oscillator by adding a bead to the middle of a stretched rubber band (Figure 19.20). The bead adds extra mass (inertia) to this simple oscillator. How would adding a bead to a rubber band change the natural frequency? Notice that gravity is not directly involved in the back and forth movement of this oscillator.

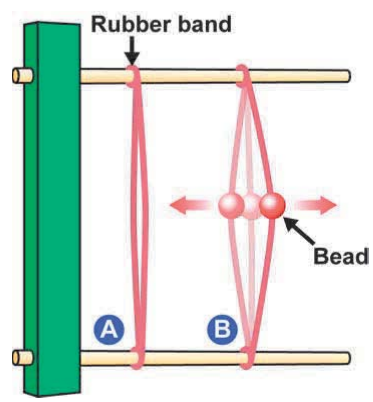

Figure 19.20: *A stretched rubber band is a good example of a vibrating string (A). You can modify this simple oscillator by adding a bead to the rubber band (B).*

19.3 Section Review

1. Identify the restoring force for a pendulum, a mass on a spring, and a vibrating string.
2. You change the amplitude of a pendulum from 10 centimeters to 30 centimeters. How does this change affect the period of the pendulum? Justify your answer.
3. Is a person jumping on a trampoline an oscillator? Justify your response.
4. If you wanted to increase the period of a pendulum, how would you change its length?

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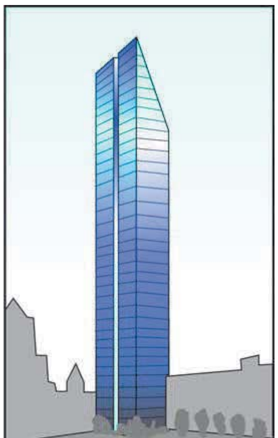
CONNECTIONS

Skyscrapers and Harmonic Motion

The John Hancock Tower is one of the tallest skyscrapers in New England. This 60-story building is 240.7 meters (790 feet) tall and was completed in 1976. With 10,344 windowpanes, the most striking feature of this building is that it is completely covered in glass!


John Hancock Tower
Boston, Massachusetts

While this skyscraper was being built in 1972 and 1973, a disaster struck—windowpanes started falling out from all over the building and crashing to the ground. So many fell out that, with the boarded up window holes, the Hancock Tower was nicknamed the "plywood palace." Some people said the windows fell out because the building swayed too much in the wind—they thought the problem was due to the natural harmonic motion of the skyscraper.

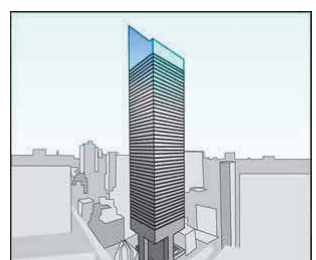
Why does a skyscraper sway?

Just like trees which experience harmonic motion in strong winds, skyscrapers also sway side to side. Skyscrapers or any buildings, even though made of steel and concrete, begin to vibrate when the wind blows or an earthquake occurs. All buildings have a fundamental frequency of vibration. For example, the fundamental frequencies for buildings range as follows: 10 hertz for one-story buildings, 2 hertz for a three- to five-story buildings, 0.5 to 1 for tall buildings (10 to 20 stories high), and 0.17 hertz for skyscrapers.

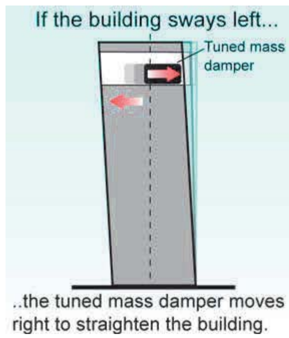
On the top floor of some skyscrapers, with a strong wind, the amplitude of their side-to-side motion ("sway") can be several feet. Therefore, engineers have carefully designed skyscrapers to handle a large swaying motion. Engineers strive to keep the amplitude very small so that the people inside will not be disturbed. When the falling windowpanes of the Hancock Tower were blamed on the building's sway, engineers were quick to point out that the John Hancock Tower was designed to sway slightly. Engineers did not think the sway of this building was causing the falling windows.

Swaying is a form of simple harmonic motion. Swaying starts with a disturbing or force such as the wind pushing on the side of the building. A restoring force keeps the motion always accelerating back towards its equilibrium point. In a skyscraper, the equilibrium point is when the building is perfectly straight. For a skyscraper, the restoring force is provided by the mass of the structure of the skyscraper. The Hancock Tower has a stiff backbone made up of steel columns and beams in the skyscraper's core. That extra sturdiness allows the building to bend slightly and then ease back towards its center point. Some skyscrapers get their restoring force from hollow, rigid tubes at the perimeter of the structure. The advantage of the tubes is that they are a strong core design, with less weight.

The Citicorp Center in New York City was the first building to have a mechanical means for providing a restoring force to counteract swaying. A 410-ton concrete weight housed on the top floors of the building slides back and forth in opposition to the sway caused by wind. Thus, the restoring force in the Citicorp Center is accomplished by


Citicorp Center
New York, New York

shifting the center of mass of the building so that gravity pulls the building back towards its "straight" or equilibrium position. The device used in the Citicorp Center is called a wind-compensating damper or "tuned mass damper."



William LeMessurier, an innovative engineer, installed the tuned mass damper in the Citicorp Center. LeMessurier was also involved in installing a tuned mass damper in the Hancock Tower. This device wasn't necessary to stop windows from falling, but was used to keep the building from twisting as it swayed — a very disturbing affect felt by the people on the top floors of the building.

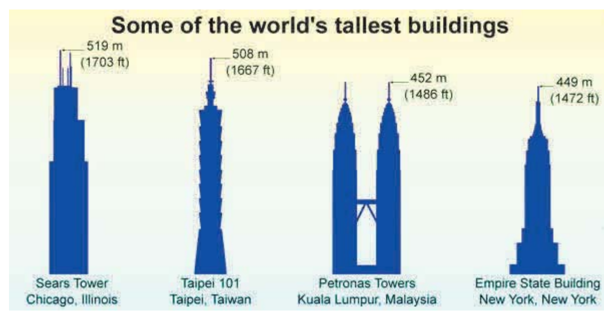
What is the tallest building in the world?

The current world champion of skyscrapers (2004) is Taipei 101 located in Taiwan. It is 508 meters tall (1,667 feet) with 101 floors above ground. Since both earthquakes and wind are concerns in Taiwan, the building's engineers took extra precautions. The 800-ton wind-compensating damper at the top of the building is a large spherical shape hung as a simple pendulum. The damper is visible to the public on the 88th and 89th floors where there is a restaurant! When the building begins to sway either due to wind or an earthquake, the damper acts as a restoring force. The Taipei 101 is built to withstand an earthquake greater than 7 on the Richter scale!

Other countries are currently constructing skyscrapers that will be even taller than Taipei 101. So, this world record holder will not be the tallest building for very long. With modern materials, and future innovations yet to come, the main limitation to the height of future skyscrapers is the cost to build such tall buildings!

The reason for the falling windows

The windows of the Hancock Tower fell out because of how the double-paned glass was bonded to the window frame. The bonding prevented the glass from responding to temperature changes and wind forces. Because the windows were held too rigidly by the bonding, the glass fractured easily and fell out. The modern John Hancock Tower sways slightly in the wind just like before, but without twisting thanks to the tuned mass damper. Also, the bonding of the windows has been fixed, and now the windows stay in place.



Questions:

1. From the reading, why were the windows falling out of the John Hancock Tower?
2. Describe the sway of a building. Use the terms force and harmonic motion in your answer.
3. Research and write a brief report about William LeMessier's work on the Citicorp Center.
4. Research the John Hancock Tower and find out what its tuned mass damper looks like and how it works.
5. Find out why Taipei 101 "beats" the Sears Tower as the world's tallest building.

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Chapter 19 Review

Understanding Vocabulary

Select the correct term to complete the sentences.

oscillators	phase	harmonic motion
frequency	resonance	cycle
natural frequency	hertz	period
damping	restoring force	vibration
amplitude		

Section 19.1

1. Frequency is measured in ____.
2. A ____ is the building block of harmonic motion and has a beginning and an end.
3. The time it takes for one cycle is called the ____.
4. Motion that repeats itself over and over is called ____.
5. The number of cycles an oscillator makes per second is called the ____.
6. A pendulum, an atom, and the solar system are all examples of ____.
7. ____ describes the size of a cycle.
8. Swinging motion (back and forth motion that repeats) is an example of a ____.
9. Friction causes ____ in an oscillator.

Section 19.2

10. The ____ of an oscillator describes where it is in the cycle.

Section 19.3

11. When the frequency of a periodic force matches the natural frequency of the oscillating system, ____ occurs.
12. A guitar is tuned by adjusting the ____ of the vibrating string to match a musical note.
13. The ____ is a force that always acts to pull an oscillator back toward the center position.

Reviewing Concepts

Section 19.1

1. Identify the following as examples of harmonic motion, linear motion, or both. Explain your answer.
   a. A child moving down a playground slide one time
   b. An ocean wave rising and falling
   c. A car moving down the street
   d. A ball bouncing up and down
2. A system with harmonic motion is called an oscillator. Oscillators can be virtually any size. List at least one example of a very large oscillator and a very small oscillator.
3. Describe a single cycle of harmonic motion for the following situations:
   a. A spinning merry-go-round
   b. Earth's orbit around the sun
   c. A clock pendulum
4. Using a person on a swing as an example of harmonic motion, describe these terms:
   a. period
   b. frequency
   c. cycle
   d. amplitude
5. Your favorite radio station is 106.7. What are the units on this number and what do they mean in terms of harmonic motion?
6. What is the mathematical relationship between frequency and period for a harmonic motion system?
7. Name a unit used to measure the following:
   a. amplitude
   b. frequency
   c. period
   d. mass

Section 19.2

8. Describe how you would determine the period and amplitude of an oscillator from a graph of its harmonic motion. You may use a diagram to help you answer this question.
9. Two players dribble basketballs at the same time. How does the motion of the basketballs compare if they are in phase? out of phase?
10. Explain why circular motion, like the motion of a ferris wheel, is an example of harmonic motion.

Section 19.3

11. If the length of the rope on a swing gets longer:
    a. What happens to the period of the swing?
    b. What happens to the frequency of the swing?
12. Pushing a child on a playground swing repeatedly at the natural frequency causes resonance, which increases the amplitude of the swing, and the child goes higher. If the pushes provide the periodic force of the system, what provides the restoring force?
13. Identify the equilibrium position for the following situations.
    a. A person on a swing
    b. A person bungee jumping
    c. A guitar string being plucked
14. What is resonance and how is it created? Give an example of a resonant oscillating system in nature.

Solving Problems

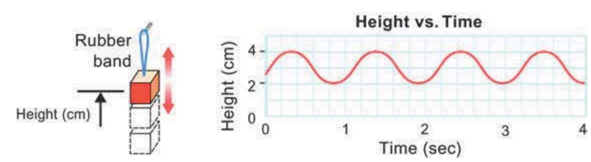
Section 19.1

1. The wings of a honeybee move at a frequency of 220 Hz. What is the period for a complete wing-beat cycle?
2. If a pendulum's period is 4 seconds, what is its frequency?
3. What is the period of Earth spinning on its axis? What is its frequency? (Hint: How long does it take for one spin?)
4. Jason's heartbeat is measured to be 65 beats per minute.
    a. What is the frequency of heartbeats in hertz?
    b. What is the period for each heartbeat in seconds?
5. In the table below, fill in the period and frequency for the second hand, minute hand, and hour hand of a clock.

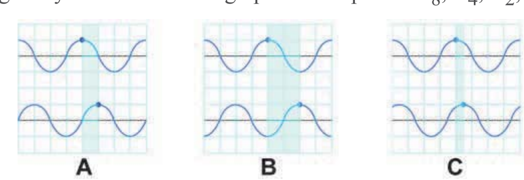
	Period (seconds)	Frequency (hertz)
Second hand		
Minute hand		
Hour hand		

Section 19.2

6. The graph shows the motion of an oscillator that is a weight hanging from a rubber band. The weight moves up and down. Answer the following questions using the graph.



    a. What is the period?
    b. What is the frequency?
    c. What is the amplitude?
    d. If you count for 5 seconds, how many cycles would you count?
7. Make a graph of three cycles of motion for a pendulum that has a period of 2 seconds and an amplitude of 5 centimeters.
8. Which of the following graphs illustrates the harmonic motion of two children on swings 180 degrees out of phase. What fraction of a 360-degree cycle are these two graphs out of phase: $^1/_8$, $^1/_4$, $^1/_2$, or $^3/_4$?



Section 19.3

9. The mass on a pendulum bob is increased by a factor of two. How is the period of the pendulum affected?

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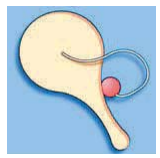
10. Describe how you might change the natural frequency of the following oscillating systems.
    a. a guitar string
    b. a playground swing
    c. a paddle ball game with a ball attached to a paddle with an elastic
    d. a diving board

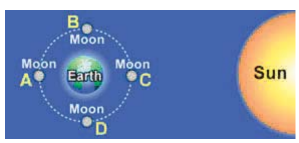


11. How does decreasing the length of a pendulum affect its period?

Applying Your Knowledge

Section 19.1

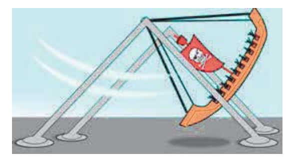
1. The human heart is both strong and reliable. As a demonstration of how reliable the heart is, calculate how many times your heart beats in one day. Start by measuring the frequency of your pulse in beats per minute and use the result for your calculation.
2. Ocean tides rise and fall based on the position of the moon as it moves around Earth. The ocean's water is pulled in the direction of the moon by the moon's gravity. The sun's gravity also affects the tides, but because of its great distance from Earth, the effect is not as strong as the moon's. The graphic shows different positions of the moon relative to Earth and the sun.



    a. Which two positions of the moon result in greater tide amplitudes (these are called *spring tides*)? Which two positions result in smaller tide amplitude (these are called *neap tides*)? Refer to the graphic above.
    b. Challenge question: In many places on Earth there are two high tides and two low tides each day. Why do you think this happens?
3. The solar system is an oscillator with each of the planets in harmonic motion around the sun. Give five examples of cycles that relate to the solar system and also give the period and frequency of each example. Use your library or the Internet to find the answers to this question.
4. A sewing machine makes sewing stitches, a repeating task, easier. As a result, many parts of a sewing machine have harmonic motion. Find a sewing machine to examine. List two parts of this machine that use harmonic motion. If you don't know the names of certain parts, make a diagram of the machine to help you explain your answer.

Section 19.2

5. The graphic shows the harmonic motion of a pirate ship amusement park ride. Use what you know about kinetic and potential energy to answer the following questions.



    a. Where in its cycle does the pirate ship have its highest potential energy? its lowest potential energy?
    b. Where in its cycle does pirate ship have its highest kinetic energy? its lowest kinetic energy?
    c. Make a graph of the amount of kinetic energy the ride has during one cycle of motion. On the same graph, plot the amount of potential energy during one cycle of motion. Use the point at which the ride it at its highest as the starting point of the cycle. Is this graph like a harmonic motion graph? Why or why not?

Section 19.3

6. Buildings are not completely stiff — they sway side-to-side at their natural frequency. What do you think happens if the natural frequency of a building matches the frequency of an earthquake? How could a building's natural frequency be changed?
7. The cycle of motion of a pendulum is created by the restoring force of the weight of the bob. As with all motion, the harmonic motion of a pendulum must follow Newton's laws of motion.
    a. Newton's first law states that objects tend to keep doing what they are doing. How does the first law apply to a pendulum?
    b. Newton's second law states that $a = F \times m$. Which fact about the motion of a pendulum does the second law explain: (1) that changing the mass *does not* change the period, or (2) that changing the length *does* change the period.
    c. Name an action-reaction pair of the pendulum that illustrates Newton's third law.