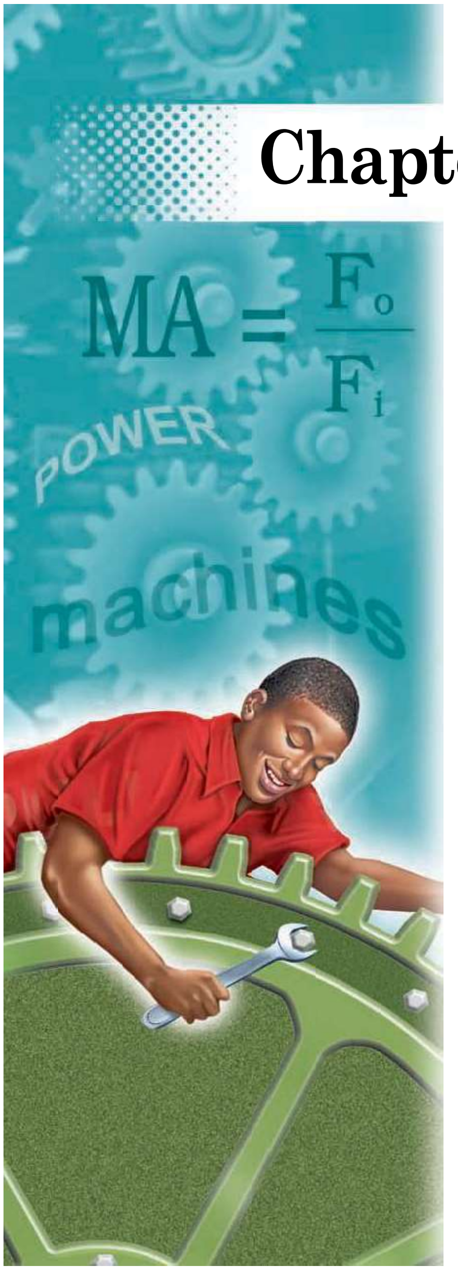
Table of Contents



Chapter 5

Forces in Equilibrium

Many people would not consider it extraordinary to get into an elevator and zoom to the top of a 50 story building. They might not be so nonchalant if they knew the balance of enormous forces that keeps a tall building standing up. Or, they might feel even more secure, knowing how well the building has been engineered to withstand the forces.

Tall buildings are an impressive example of equilibrium, or the balancing of forces. The average acceleration of a building should be zero! That means all forces acting on the building must add up to zero, including gravity, wind, and the movement of people and vehicles. A modern office tower is constructed of steel and concrete beams that are carefully designed to provide reactions forces to balance against wind, gravity, people, and vehicles.

In ancient times people learned about equilibrium through trial-and-error. Then, as today, different builders and architects each wanted to make a building taller than the others. Without today's knowledge of equilibrium and forces, many builders experimented with designs that quickly fell down. It is estimated that ten cathedrals fell down for every one that is still standing today! Over time, humans learned the laws of forces and equilibrium that allow us to be much more confident about the structural strength of modern tall buildings.

Key Questions

- ✓ How do you precisely describe a force?
- ✓ How is the concept of equilibrium important to the design of buildings and bridges?
- ✓ What is friction?
- ✓ How is torque different from force?

Table of Contents

5.1 The Force Vector

Think about how to accurately describe a force. One important piece of information is the strength of the force. For example, 50 newtons would be a clear description of the strength of a force. But what about the direction? The direction of a force is important, too. How do you describe the direction of a force in a way that is precise enough to use for physics? In this section you will learn that force is a *vector*. A vector is a quantity that includes information about both size (strength) and direction.

Vocabulary

scalar, magnitude, vector, component, free body diagram

Objectives

✓ Draw vectors to scale to represent a quantity's magnitude and direction.
✓ Solve vector problems.
✓ Find a vector's components.

Scalars and vectors

Scalars have magnitude

A **scalar** is a quantity that can be completely described by a single value called **magnitude**. Magnitude means the size or amount and always includes units of measurement. Temperature is a good example of a scalar quantity. If you are sick and use a thermometer to measure your temperature, it might show 101°F. The magnitude of your temperature is 101, and degrees Fahrenheit is the unit of measurement. The value of 101°F is a complete description of the temperature because you do not need any more information.

Examples of scalars

Many other measurements are expressed as scalar quantities. Distance, time, and speed are all scalars because all three can be completely described with a single number and a unit.

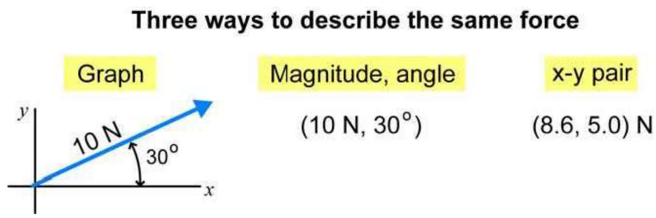
Vectors have direction

Sometimes a single number does not include enough information to describe a measurement. In giving someone directions to your house, you could not tell him simply to start at his house and drive four kilometers. A single distance measurement is not enough to describe the path the person must follow. Giving complete directions would mean including instructions to go two kilometers to the north, turn right, then go two kilometers to the east (Figure 5.1). The information "two kilometers to the north" is an example of a **vector**. A vector is a quantity that includes both magnitude and direction. Other examples of vectors are force, velocity, and acceleration. Direction is important to fully describe each of these quantities.

Figure 5.1: *Vectors are useful in giving directions.*

The force vector

What is a force vector? A force vector has units of newtons, just like all forces. In addition, the force vector also includes enough information to tell the direction of the force. There are three ways commonly used to represent both the strength and direction information: a graph, an *x-y* pair, and a magnitude-angle pair. You will learn all three in this chapter because each is useful in a different way.



Drawing a force vector The graph form of the force vector is a picture showing the strength and direction of a force. It is just like an ordinary graph except the *x*- and *y*-axes show the strength of the force in the *x* and *y* directions. The force vector is drawn as an arrow. The length of the arrow shows the magnitude of the vector, and the arrow points in the direction of the vector.

Scale When drawing a vector, you must choose a scale. A scale for a vector diagram is similar to a scale on any graph. For example, if you are drawing a vector showing a force of five newtons pointing straight up (y-direction) you might use a scale of one centimeter to one newton. You would draw the arrow five centimeters long pointing along the y-direction on your paper (Figure 5.2). You should always state the scale you use when drawing vectors.

***x* and *y* forces** When you draw a force vector on a graph, distance along the *x*- or *y*-axes represents the strength of the force in the *x*- and *y*-directions. A force at an angle has the same effect as two smaller forces aligned with the *x*- and *y*-directions. As shown in Figure 5.3, the 8.6-newton and 5-newton forces applied together have the exact same effect as a single 10-newton force applied at 30 degrees. This idea of breaking one force down into an equivalent pair of *x*- and *y*-forces is very important, as you will see.

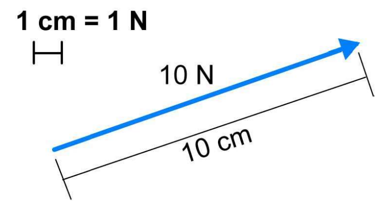


Figure 5.2: *A 10-newton force vector, with a scale of one centimeter to one newton.*

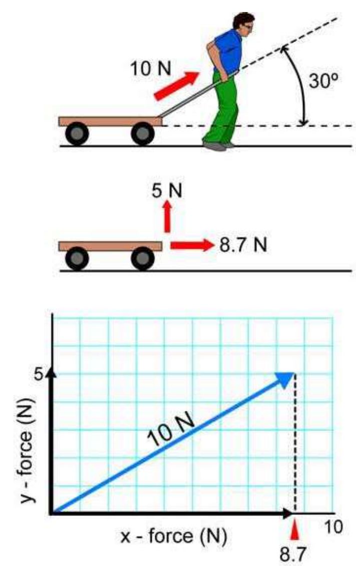


Figure 5.3: *A force at an angle has the same effect as two smaller forces applied at the same time along the x- and y- directions.*

Table of Contents

Vector components

Components Every force vector can be replaced by perpendicular vectors called **components**. You can think of components as adding up to make the original force. When adding and subtracting forces, it is usually much easier to work with the components than it is with the original force.

Finding components Figure 5.4 shows how to find the components of a force vector using a graph. There are three steps. The first step is to draw the force vector to scale and at the correct angle. Second, extend lines parallel to the *x*- and *y*-axes. Third, read off the *x*- and *y*-components from the scales on the *x*- and *y*-axes. In the example, the *x*-component is 8.6 newtons, the *y*-component 5 newtons.

Using a triangle Another way to find the components of a force vector is to make a triangle (Figure 5.5). The *x*- and *y*-components are the lengths of the sides of the triangle parallel to the *x*- and *y*-axes. You can check your work with the Pythagorean theorem. The components are the legs of the triangle, or sides *a* and *b*. The original vector is the hypotenuse, or side *c*. According to the Pythagorean theorem $a^2 + b^2 = c^2$. In terms of the forces in the example, this means $(5\text{ N})^2 + (8.6\text{ N})^2 = (10\text{ N})^2$.

Writing an (x, y) vector If you know the *x*- and *y*-components you can write a force vector with parentheses. The force in Figure 5.4 is written (8.6, 5) N. The first number is the *x*-component of the force, the second number the *y*-component. It is much easier to add or subtract forces when they are in *x*- and *y*-components. Mathematically, when we write a vector as (x, y) we are using *cartesian coordinates*. Cartesian coordinates use perpendicular *x*- and *y*- axes like graph paper.

Polar coordinates The third way to write a force vector is with its magnitude and angle. The force in Figure 5.4 is (10 N, 30°). The first number (10 N) is the magnitude, or strength of the force. The second number is the angle measured from the *x*-axis going counterclockwise. Mathematically, this way of writing a vector is in *polar coordinates*.

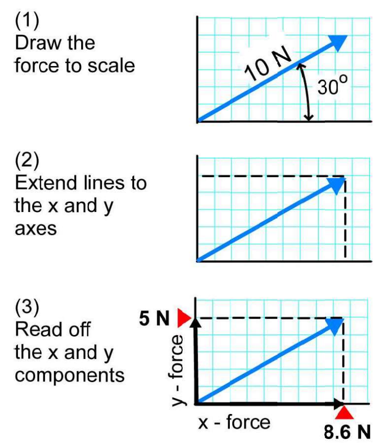
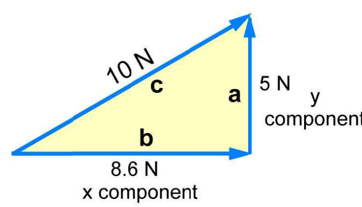


Figure 5.4: *Finding the components of a 10-newton force vector at a 30-degree angle.*



Pythagorean theorem

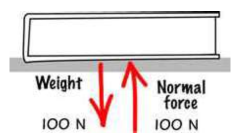
$$a^2 + b^2 = c^2$$

Figure 5.5: *Finding the components using a triangle.*

Free-body diagrams

Drawing free-body diagrams

A **free-body diagram** is a valuable tool used to study forces. It is a diagram that uses vectors to show all of the forces acting on an object. The free-body diagram for a book sitting on a table is shown in to the right. A free-body diagram shows only the forces acting *on* an object, and does not include the forces an object exerts on other things. When making a free-body diagram, draw only the object you are studying, not any other objects around it. Be sure to clearly label the strength of the force shown by each vector.



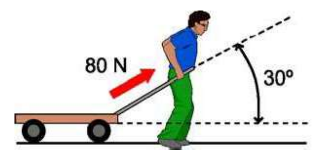


Figure 5.6: *Find the x and y components of the force.*

Components of a force

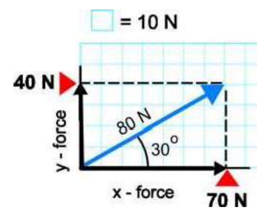
A man pulls a wagon with a force of 80 N at an angle of 30 degrees. Find the *x* (horizontal) and *y* (vertical) components of the force (Figure 5.6).

1. Looking for: You are asked for the *x*- and *y*-components of the force.

2. Given: You are given the magnitude and direction of the force.

3. Relationships: The *x*- and *y*-components can found by graphing the force.

4. Solution: *x*-component is 70 N, *y*-component is 40 N (see diagram at right).



Your turn...

a. What is the vertical (*y*) component of a 100-newton force at an angle of 60 degrees to the *x*-axis? **Answer:** 86.6 N

b. Two people push on a heavy box. One pushes with a force of 100 newtons toward 90°, and the other pushes with a force of 70 newtons toward 180°. Use a scaled drawing (1 cm = 10 N) to find the net force. **Answer:** 122 N

5.1 Section Review

1. What is the difference between a scalar and a vector?
2. Is each of these a scalar or a vector: speed, time, mass, weight, velocity, temperature.
3. Draw a force vector to scale that represents a force of 200 N at 120°.
4. Draw the force vector (6, 8) N. Is this the same as the force vector (100 N, 53°)?
5. A ball is hanging straight down on a string. Draw a free body diagram of the ball.

Table of Contents

5.2 Forces and equilibrium

Sometimes you want things to accelerate and sometimes you don't. Cars should accelerate, bridges should not. In order for a bridge to stay in place, *all* the forces acting on the bridge must add up to produce zero net force. This section is about *equilibrium*, which is what physicists call any situation where the net force is zero. The concept of equilibrium is important to the design of buildings, bridges, and virtually every technology ever invented by humans.

Vocabulary

equilibrium, normal force, resultant, Hooke's law, spring constant

Objectives

- ✓ Explain what it means to say an object is in equilibrium.
- ✓ Use free-body diagrams to find unknown forces.
- ✓ Explain how springs exert forces.
- ✓ Add force vectors.

Equilibrium

Definition of equilibrium The net force on an object is the vector sum of all the forces acting on it. When the net force on an object is zero, we say the object is in **equilibrium**. Newton's first law says an object's motion does not change unless a net force acts on it. If the net force is zero (equilibrium), an object at rest will stay at rest and an object in motion will stay in motion with constant speed and direction.

The second law The second law says the acceleration of an object in equilibrium is zero because the net force acting on the object is zero. Zero acceleration means neither the speed nor the direction of motion can change.

Normal force Any object at rest is in equilibrium and has a net force of zero acting on it. Imagine a book sitting on a table. Gravity pulls the book downward with a force equal to the book's weight. But what force balances the weight? The table exerts an upward force on the book called the **normal force**. The word normal here has a different meaning from what you might expect. In mathematics, normal means *perpendicular*. The force the table exerts is perpendicular to the table's surface.

Newton's third law Newton's third law explains why normal forces exist (Figure 5.7). The book pushes down on the table, so the table pushes up on the book. The book's force on the table is the action force, and the table's force on the book is the reaction force. The third law says that these forces are equal in strength. If the book is at rest, these forces *must* be equal but opposite in direction. If the book were heavier, it would exert a stronger downward force on the table. The table would then exert a stronger upward force on the book.

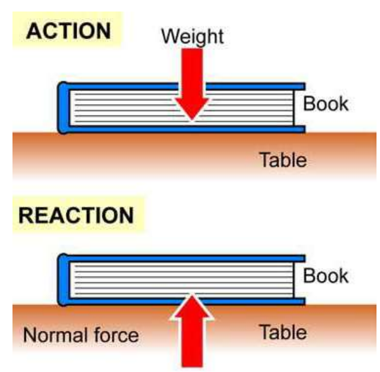


Figure 5.7: *The book pushes down on the table, and the table pushes up on the book. The force exerted by the table is called the normal force.*

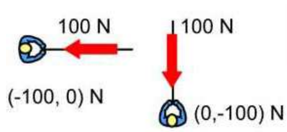
Adding force vectors

An example Suppose three people are trying to keep an injured polar bear in one place. Each person has a long rope attached to the bear. Two people pull on the bear with forces of 100 N each (Figure 5.8). What force must the third person apply to balance the other two? The bear will not move if the net force is zero. To find the answer, we need to find the net force when the forces are not in the same direction. Mathematically speaking, we need a way to *add vectors*.

Graphically adding vectors On a graph you add vectors by drawing them end-to-end on a single sheet. The beginning of one vector starts at the end of the previous one. The total of all the vectors is called the **resultant.** The resultant starts at the origin and ends at the end of the last vector in the chain (Figure 5.9). The resultant in the example is a single 141 newton force at 225 degrees. To cancel this force, the third person must pull with an equal 141 N force in the opposite direction (45°). Adding force vectors this way is tedious because you must carefully draw each one to scale and at the proper angle.

Adding x-y components Adding vectors in x-y components is much easier. The *x*-component of the resultant is the sum of the *x*-components of each individual vector. The *y*-component of the resultant is the sum of the *y*-components of each individual vector. For the example, (-100, 0) N + (0, -100) N = (-100, -100) N. The components are negative because the forces point in the negative-x and negative-y directions. The resultant vector is (-100, -100) N.

Equilibrium To have zero net force, the forces in both the *x* and *y* directions must be zero. The third force must have *x* and *y* components that add up to zero when combined with the other forces. The solution to the problem is written below.



Problem
(-100, 0) N + (0, -100) N + (?, ?) N = (0,0) N
Solution
(?, ?) N = (100, 100) N

Following the rules we just gave, the third force must be (100, 100) N. This is the same as a force of 141 N at 45°.

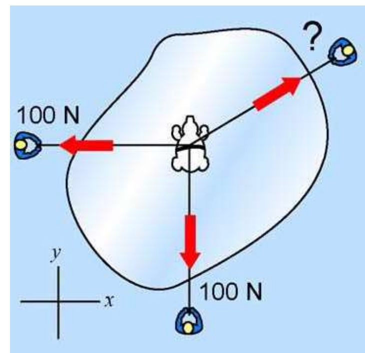


Figure 5.8: *Three people trying to keep a polar bear in the center of an ice floe.*

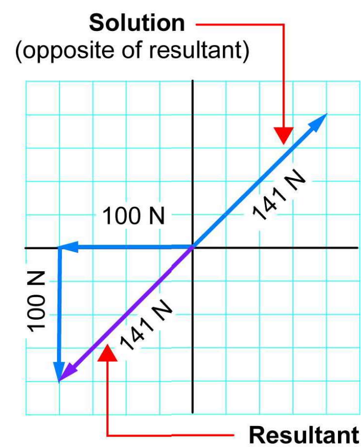


Figure 5.9: *Finding the resultant and solving the problem graphically.*

Table of Contents

Solving equilibrium problems

Finding the net force For an object to be in equilibrium, all the forces acting *on the object* must total to zero. In many problems you will need the third law to find reaction forces (such as normal forces) that act on an object.

Using vectors In equilibrium, the net force *in each direction* must be zero. That means the total force in the x-direction must be zero and total force in the y-direction also must be zero. You cannot mix x- and y-components when adding forces. Getting the forces in each direction to cancel separately is easiest to do when all forces are expressed in x-y components. Note: In three dimensions, there also will be a z-component force.

Balancing forces If you are trying to find an unknown force on an object in equilibrium, the first step is always to draw a free-body diagram. Then use the fact that the net force is zero to find the unknown force. To be in equilibrium, forces must balance both horizontally and vertically. Forces to the right must balance forces to the left, and upward forces must balance downward forces.

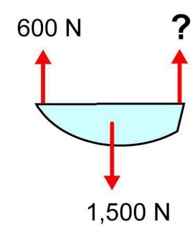


Figure 5.10: *The free-body diagram for the boat in the example problem.*

Equilibrium

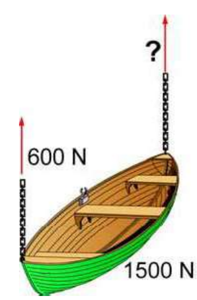
Two chains are used to lift a small boat weighing 1500 newtons. As the boat moves upward at a constant speed, one chain pulls up on the boat with a force of 600 newtons. What is the force exerted by the other chain?

1. Looking for: You are asked for an unknown force exerted by a chain.

2. Given: You are given the boat's weight in newtons and the force of one chain in newtons.

3. Relationships: The net force on the boat is zero.

4. Solution: Draw a free-body diagram (Figure 5.10).
The force of the two chains must balance the boat's weight.
$600\text{ N} + F_{chain2} = 1500\text{ N}$ $\quad F_{chain2} = 900\text{ N}$



Your turn...

a. A heavy box weighing 1000 newtons sits on the floor. You lift upward on the box with a force of 450 newtons, but the box does not move. What is the normal force on the box while you are lifting? **Answer:** 550 newtons

b. A 40-newton cat stands on a chair. If the normal force on each of the cat's back feet is 12 newtons, what is the normal force on each front foot? (You can assume it is the same on each.) **Answer:** 8 newtons

The force from a spring

Uses for springs Springs are used in many devices to keep objects in equilibrium or cause acceleration. Toasters use springs to pop up the toast, cars use springs in their suspension, and retractable pens use springs to move the pen's tip. Springs can also be used as a way to store energy. When you push the handle down on a toaster, potential energy is stored in the spring. Releasing the spring causes the potential energy to convert into kinetic energy as the toast pops up.

Stretching and compressing a spring The most common type of spring is a coil of metal or plastic that creates a force when you stretch it or compress it. The force created by stretching or compressing a spring always acts to return the spring to its natural length. When you stretch a spring, it *pulls* back on your hand as the spring tries to return to its original length. When you compress a spring and make it shorter, it *pushes* on your hand as it tries to return to its original length.

Newton's third law Newton's third law explains why a spring's force acts opposite the direction it is stretched or compressed. The top spring in Figure 5.11 stretches when you apply a force to the right. The force of your hand on the spring is the action force. The spring applies a reaction force to the left on your hand.

The bottom picture shows what happens when the spring is compressed. You must exert an action force to the left to compress the spring. The spring exerts a reaction force to the right against your hand. In both cases, the spring's force tries to return it to its original length.

Normal force and springs How does a table "know" how much normal force to supply to keep a book at rest? A table cannot solve physics problems! The answer is that the normal force exerted by a surface is very similar to the force exerted by a spring in compression (Figure 5.12). When a book sits on a table, it exerts a downward force that compresses the table's top a tiny amount. The tabletop exerts an upward force on the book and tries to return to its natural thickness. The matter in the table acts like a collection of very stiff compressed springs. The amount of compression is so small you cannot see it, but it can be measured with sensitive instruments.

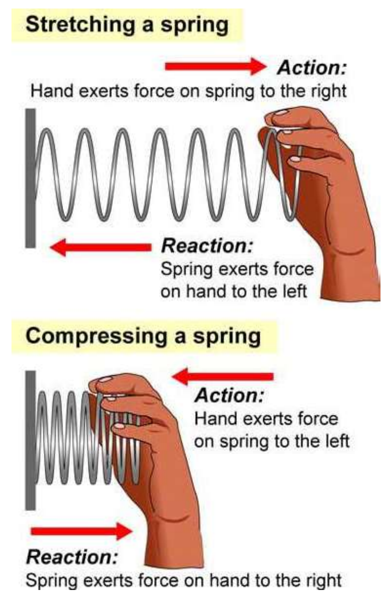


Figure 5.11: *The direction of the force exerted by the spring is opposite the direction of the force exerted by the person.*

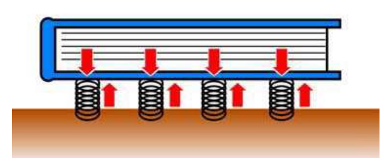


Figure 5.12: *The normal force exerted by a surface is similar to the force exerted by a compressed spring.*

Table of Contents

Hooke's law

Hooke's law The relationship between a spring's change in length and the force it exerts is called **Hooke's law**. The law states that the force exerted by a spring is proportional to its change in length. For example, suppose a spring exerts a force of five newtons when it is stretched two centimeters. That spring will exert a force of 10 newtons when it is stretched four centimeters. Doubling the stretching distance doubles the force.

Spring constant Some springs exert small forces and are easy to stretch. Other springs exert strong forces and are hard to stretch. The relationship between the force exerted by a spring and its change in length is called its **spring constant**. A large spring constant means the spring is hard to stretch or compress and exerts strong forces when its length changes. A spring with a small spring constant is easy to stretch or compress and exerts weak forces. The springs in automobile shock absorbers are stiff because they have a large spring constant. A retractable pen's spring has a small spring constant.

How scales work The relationship between force and change in length is used in scales (Figure 5.13). When a hanging scale weighs an object, the distance the spring stretches is proportional to the object's weight. An object that is twice as heavy changes the spring's length twice as much. The scale is calibrated using an object of a known weight. The force amounts are then marked on the scale at different distances. A bathroom scale works similarly but uses a spring in compression. The greater the person's weight, the more the spring compresses.

5.2 Section Review

1. Can a moving object be in equilibrium? Explain.
2. Draw a free-body diagram of a 700-newton person sitting on a chair in equilibrium.
3. The spring in a scale stretches 1 centimeter when a 5-newton object hangs from it. How much does an object weigh if it stretches the spring 2 centimeters?
4. How is normal force similar to the force of a spring?

HOOKE'S LAW

Force (newtons)

$$F = -kx$$

Spring constant (newtons/meters)

Extension or compression (meters)

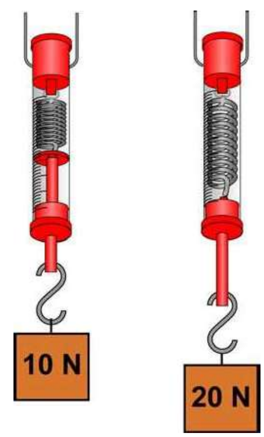


Figure 5.13: *A hanging scale uses a spring to measure weight.*

5.3 Friction

Friction forces are constantly acting on you and the objects around you. When you are riding a bicycle and just coasting along, friction is what finally slows you down. But did you know that friction also helps you to speed up? Tires need friction to push against the road and create the reaction forces that move you forward. In this section you will learn about different types of friction, the cause of friction, and how it affects the motion of objects. You will also find out how friction is useful to us and learn how to reduce it when it's not.

Vocabulary

friction, sliding friction, static friction, lubricant

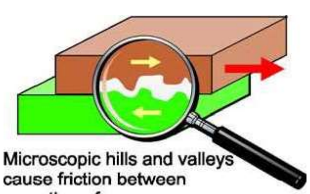
Objectives

- ✓ Distinguish between sliding and static friction.
- ✓ Explain the cause of friction.
- ✓ Discuss reasons to increase or decrease friction.

What is friction?

What is friction? **Friction** is a force that resists the motion of objects or surfaces. You feel the effects of friction when you swim, ride in a car, walk, and even when you sit in a chair. Because friction exists in many different situations, it is classified into several types (Figure 5.14). This section will focus on sliding friction and static friction. **Sliding friction** is present when two objects or surfaces slide across each other. **Static friction** exists when forces are acting to cause an object to move but friction is keeping the object from moving.

The cause of friction If you looked at a piece of wood, plastic, or paper through a powerful microscope, you would see microscopic hills and valleys on the surface. As surfaces slide (or try to slide) across each other, the hills and valleys grind against each other and cause friction. Contact between the surfaces can cause the tiny bumps to change shape or wear away. If you rub sandpaper on a piece of wood, friction affects the wood's surface and makes it either smoother (bumps wear away) or rougher (they change shape).



Microscopic hills and valleys cause friction between smooth surfaces

Two surfaces are involved Friction depends on *both* of the surfaces in contact. The force of friction on a rubber hockey puck is very small when it is sliding on ice. But the same hockey puck sliding on a piece of sandpaper feels a large friction force. When the hockey puck slides on ice, a thin layer of water between the rubber and the ice allows the puck to slide easily. Water and other liquids such as oil can greatly reduce the friction between surfaces.

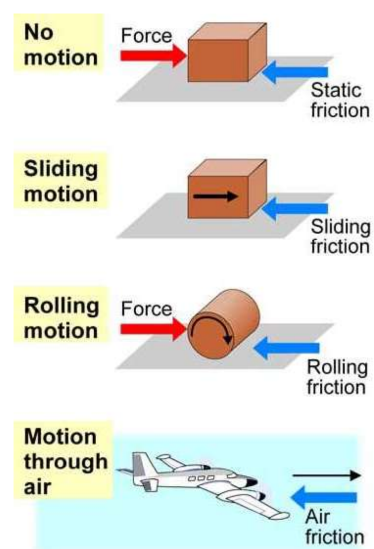


Figure 5.14: *There are many types of friction.*

Table of Contents

Identifying friction forces

Direction of the friction force Friction is a force, measured in newtons just like any other force. You draw the force of friction as another arrow on a free-body diagram. To figure out the direction of friction, always remember that friction is a *resistive* force. The force of friction acting *on* a surface always points opposite the direction of motion *of that surface*. Imagine pushing a heavy box across the floor (Figure 5.15). If you push to the right, the sliding friction acts to the left on the surface of the box touching the floor. If you push the box to the left, the force of sliding friction acts to the right. This is what we mean by saying friction resists motion.

Static friction Static friction acts to keep an object at rest from starting to move. Think about trying to push a heavy box with too small a force. The box stays at rest, therefore the net force is zero. That means the force of static friction is equal and opposite to the force you apply. As you increase the strength of your push, the static friction also increases, so the box stays at rest. Eventually your force becomes stronger than the maximum possible static friction force and the box starts to move (Figure 5.16). The force of static friction is equal and opposite your applied force up to a limit. The limit depends on details such as the types of surface and the forces between them.

Sliding friction Sliding friction is a force that resists the motion of an object already moving. If you were to stop pushing a moving box, sliding friction would slow the box to a stop. To keep a box moving at constant speed you must push with a force equal to the force of sliding friction. This is because motion at constant speed means zero acceleration and therefore zero net force. Pushing a box across the floor at constant speed is actually another example of *equilibrium*. In this case the equilibrium is created because the force you apply cancels with the force of sliding friction.

Comparing static and sliding friction How does sliding friction compare with the static friction? If you have ever tried to move a heavy sofa or refrigerator, you probably know the answer. It is harder to get something moving than it is to keep it moving. The reason is that static friction is greater than sliding friction for almost all combinations of surfaces.

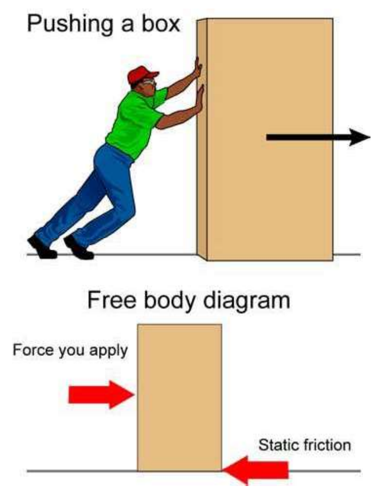


Figure 5.15: *The direction of friction is opposite the direction the box is pushed.*

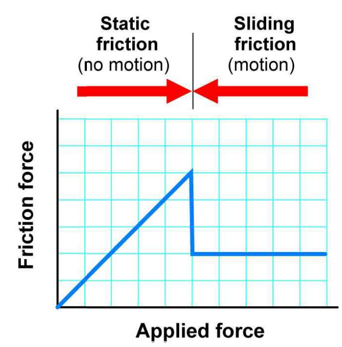


Figure 5.16: *How the friction forces on the box change with the applied force.*

A model for friction

Different amounts of friction The amount of friction that exists when a box is pushed across a smooth floor is greatly different from when it is pushed across a carpeted floor. Every combination of surfaces produces a unique amount of friction depending upon types of materials, degrees of roughness, presence of dirt or oil, and other factors. Even the friction between two identical surfaces changes as the surfaces are polished by sliding across each other. No one model or formula can accurately describe the many processes that create friction. Even so, some simple approximations are useful.

An example Suppose you pull a piece of paper across a table. To pull the paper at a constant speed, the force you apply must be equal in strength to the sliding friction. It is easy to pull the paper across the top of the table because the friction force is so small; the paper slides smoothly. Do you believe the friction force between the paper and the table is a value that cannot be changed? How might you test this question?

Friction and the force between surfaces Suppose you place a brick on the piece of paper (Figure 5.17). The paper becomes much harder to slide. You must exert a greater force to keep the paper moving. The two surfaces in contact are still the paper and the tabletop, so why does the brick have an effect? The brick causes the paper to press harder into the table's surface. The tiny hills and valleys in the paper and in the tabletop are pressed together with a much greater force, so the friction increases.

The greater the force squeezing two surfaces together, the greater the friction force.

The friction force between two surfaces is approximately proportional to the force the surfaces exert on each other. The greater the force squeezing the two surfaces together, the greater the friction force. This is why it is hard to slide a heavy box across a floor. The force between the bottom of the box and the floor is the weight of the box. Therefore, the force of friction is also proportional to the weight of the box. If the weight doubles, the force of friction also doubles. Friction is present between all sliding surfaces.

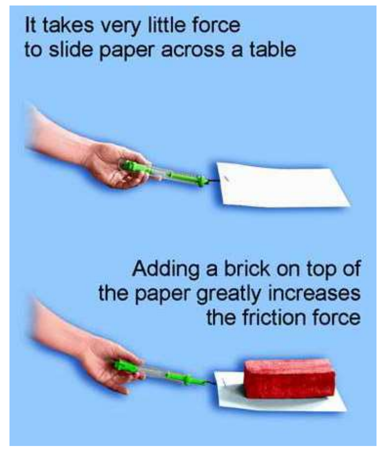


Figure 5.17: *Friction increases greatly when a brick is placed on the paper.*

Table of Contents

Reducing the force of friction

All surfaces experience some friction

Any motion where surfaces move across each other or through air or water always creates some friction. Unless a force is applied continually, friction will slow all motion to a stop eventually. For example, bicycles have low friction, but even the best bicycle slows down if you coast on a level road. Friction cannot be eliminated, though it can be reduced.

Lubricants reduce friction in machines

Keeping a fluid such as oil between two sliding surfaces keeps them from touching each other. The tiny hills and valleys don't become locked together, nor do they wear each other away during motion. The force of friction is greatly reduced, and surfaces do not wear out as fast. A fluid used to reduce friction is called a **lubricant**. You add oil to a car engine so that the pistons will slide back and forth with less friction. Even water can be used as a lubricant under conditions where there is not too much heat. A common use of powdered graphite, another lubricant, is in locks; spraying it into a lock helps a key work more easily.

Ball bearings

In systems where there are rotating objects, ball bearings are used to reduce friction. Ball bearings change sliding motion into rolling motion, which has much less friction. For example, a metal shaft rotating in a hole rubs and generates a great amount of friction. Ball bearings that go between the shaft and the inside surface of the hole allow it to spin more easily. The shaft rolls on the bearings instead of rubbing against the walls of the hole. Well-oiled bearings rotate easily and greatly reduce friction (Figure 5.18).

Magnetic levitation

Another method of reducing friction is to separate the two surfaces with a cushion of air. A hovercraft floats on a cushion of air created by a large fan. Magnetic forces can also be used to separate surfaces. A magnetically levitated (or maglev) train uses magnets that run on electricity to float on the track once the train is moving (Figure 5.19). Because there is no contact between train and track, there is far less friction than with a standard train on tracks. The ride is smoother, allowing for much faster speeds. Maglev trains are not yet in wide use because they are much more expensive to build than regular trains. They may yet become popular in the future.

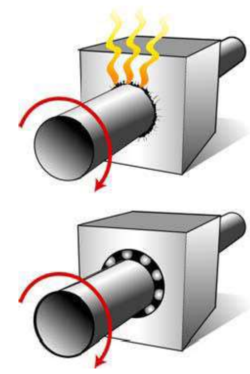


Figure 5.18: *The friction between a shaft (the long pole in the picture) and an outer part of a machine produces a lot of heat. Friction can be reduced by placing ball bearings between the shaft and the outer part.*

Figure 5.19: *With a maglev train, there is no contact between the moving train and the rail — and thus little friction.*

Using friction

Friction is useful for brakes and tires

There are many applications where friction is both useful and necessary. For example, the brakes on a bicycle create friction between two rubber *brake pads* and the rim of the wheel. Friction between the brake pads and the rim slows the bicycle. Friction is also necessary to make a bicycle go. Without friction, the bicycle's tires would not grip the road.

Weather condition tires

Rain and snow act like lubricants to separate tires from the road. As a tire rolls over a wet road, the rubber squeezes the water out of the way so that there can be good contact between rubber and road surface. Tire treads have grooves that allow space for water to be channeled away where the tire touches the road (Figure 5.20). Special irregular groove patterns, along with tiny slits, have been used on snow tires to increase traction in snow. These tires keep snow from getting packed into the treads and the design allows the tire to slightly change shape to grip the uneven surface of a snow-covered road.

Nails

Friction is the force that keeps nails in place (Figure 5.21). The material the nail is hammered into, such as wood, pushes against the nail from all sides. Each hit of the hammer drives the nail deeper into the wood, increasing the length of the nail being compressed. The strong compression force creates a large static friction force and holds the nail in place.

Cleated shoes

Shoes are designed to increase the friction between their soles and the ground. Many types of athletes, including football and soccer players, wear shoes with cleats that increase friction. Cleats are projections like teeth on the bottom of the shoe that dig into the ground. Players wearing cleats can exert much greater forces against the ground to accelerate and to keep from slipping.

5.3 Section Review

1. Explain the causes of sliding friction and static friction.
2. What do you know about the friction force on an object pulled at a constant speed?
3. What factors affect the friction force between two surfaces?
4. Give an example of friction that is useful and one that is not useful. Use examples not mentioned in the book.

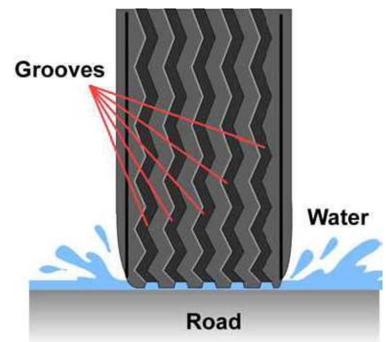


Figure 5.20: *Grooved tire treads allow space for water to be channeled away from the road-tire contact point, allowing for more friction in wet conditions.*

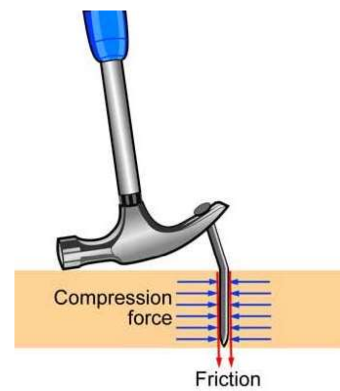
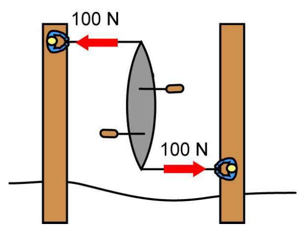


Figure 5.21: *Friction is what makes nails hard to pull out and gives them the strength to hold things together.*

Table of Contents

5.4 Torque and Rotational Equilibrium

A canoe is gliding between two docks. On each dock is a person with a rope attached to either end of the canoe. Both people pull with equal and opposite force of 100 newtons so that the net force on the canoe is zero. What happens to the canoe? It is *not* in equilibrium even though the net force is zero. The canoe rotates around its center! The canoe rotates because it is not in *rotational* equilibrium even though it *is* in force equilibrium. In this section you will learn about torque and rotational equilibrium.



Vocabulary

torque, rotate, axis of rotation, line of action, lever arm, rotational equilibrium

Objectives

✓ Explain how torque is created.
✓ Calculate the torque on an object.
✓ Define rotational equilibrium.

What is torque?

Torque and force **Torque** is a new action created by forces that are applied off-center to an object. Torque is what causes objects to **rotate** or spin. Torque is the rotational equivalent of force. If force is a *push* or *pull*, you should think of torque as a *twist*.

The axis of rotation The line about which an object turns is its **axis of rotation.** Some objects have a fixed axis: a door's axis is fixed at the hinges. A wheel on a bicycle is fixed at the axle in the center. Other objects do not have a fixed axis. The axis of rotation of a tumbling gymnast depends on her body position.

The line of action Torque is created whenever the **line of action** of a force does not pass through the axis of rotation. The line of action is an imaginary line in the direction of the force and passing through the point where the force is applied. If the line of action passes through the axis the torque is *zero*, no matter how strong a force is used!

Creating torque A force creates more torque when its line of action is far from an object's axis of rotation. Doorknobs are positioned far from the hinges to provide the greatest amount of torque (Figure 5.22). A force applied to the knob will easily open a door because the line of action of the force is the width of the door away from the hinges. The same force applied to the hinge side of the door does nothing because the line of action passes through the axis of rotation. The first force creates torque while the second does not.

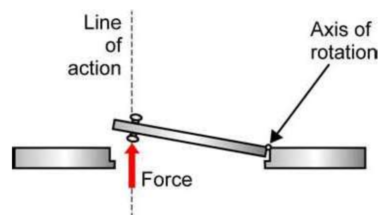


Figure 5.22: *A door rotates around its hinges and a force creates the greatest torque when the force is applied far from the hinges.*

The torque created by a force

Calculating torque The torque created by a force depends on the strength of the force and also on the **lever arm**. The lever arm is the perpendicular distance between the line of action of the force and the axis of rotation (Figure 5.23). Torque is calculated by multiplying the force and the lever arm. The Greek letter "tao" (τ) is used to represent torque; the lever arm is represented with a lower-case *r* from the word radius; and force, remember, is an upper-case F.

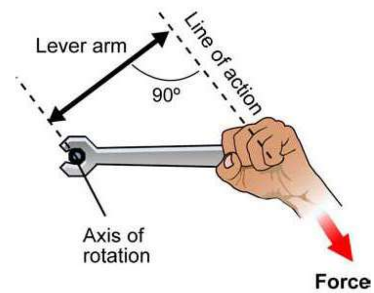
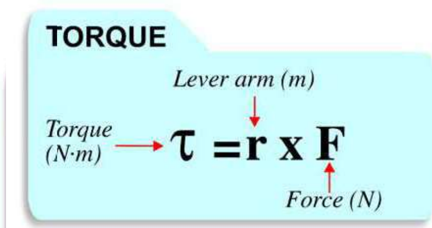
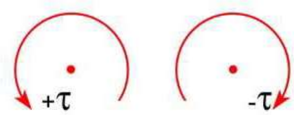


Figure 5.23: *The lever arm is the perpendicular distance between the line of action of the force and the axis of rotation.*





Showing torque in diagrams

Direction of torque The direction of torque is often drawn with a circular arrow showing how the object would rotate. The words *clockwise* and *counterclockwise* are also used to specify the direction of a torque.

Units of torque When force is in newtons and distance is in meters, the torque is measured in newton·meters (N·m). To create one newton·meter of torque, you can apply a force of one newton to a point one meter away from the axis. A force of only one-half newton applied two meters from the axis creates the *same* torque.

How torque and force differ Torque is created by force but is not the same thing as force. Torque depends on both force and distance. Torque (N·m) has different units from force (N). Finally, the same force can produce *any* amount of torque (including zero) depending on where it is applied (Figure 5.24).

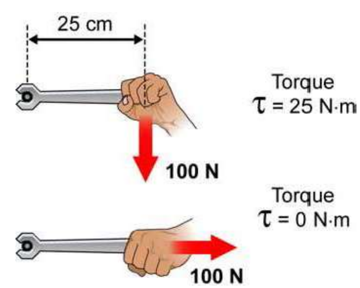


Figure 5.24: *The same force can create different amounts of torque depending on where it is applied and in what direction.*

Torque is not work The newton·meter used for torque is *not* the same as the newton·meter for work, and is *not* equal to a joule. Work is done when a force *moves* an object a distance in the direction of the force. The distance that appears in torque is the distance away from the axis of rotation. The object does not move in *this* direction. The force that creates torque causes no motion in *this* direction, so no work is done.

Table of Contents

Solving problems with torque

Reaction torque If you push *up* on a doorknob, you create a torque that tries to rotate the door upward instead of around its hinges. Your force *does* create a torque, but the hinges stop the door from rotating this way. The hinges exert reaction forces on the door that create torques in the direction opposite the torque you apply. This reaction torque is similar to the normal force created when an object presses down on a surface.

Combining torques If more than one torque acts on an object, the torques are combined to determine the net torque. Calculating net torque is very similar to calculating net force. If the torques tend to make an object spin in the same direction (clockwise or counterclockwise), they are added together. If the torques tend to make the object spin in opposite directions, the torques are subtracted (Figure 5.25).

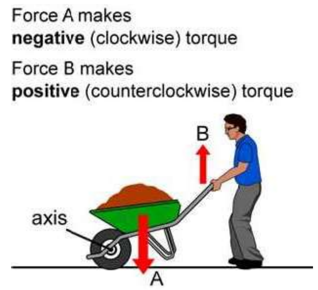
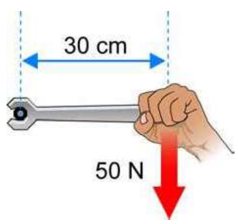


Figure 5.25: *Torques can be added and subtracted.*

Calculating torque



A force of 50 newtons is applied to a wrench that is 0.30 meters long. Calculate the torque if the force is applied perpendicular to the wrench at left.

1. Looking for: You are asked for the torque.

2. Given: You are given the force in newtons and the length of the lever arm in centimeters.

3. Relationships: Use the formula for torque, $\tau = rF$.

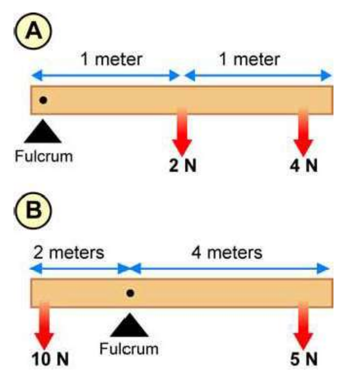
4. Solution: $\tau = (0.30 \text{ m})(50 \text{ N}) = 15 \text{ N·m}$

Your turn...

a. You apply a force of 10 newtons to a doorknob that is 0.80 meters away from the edge of the door on the hinges. If the direction of your force is straight into the door, what torque do you create? **Answer**: 8 N·m

b. Calculate the net torque in diagram A (at right). **Answer:** 10 N·m

c. Calculate the net force and the net torque in diagram B (at right). **Answer:** 5 N and 0 N·m



Rotational equilibrium

Rotational equilibrium

An object is in **rotational equilibrium** when the net torque applied to it is zero. For example, if an object such as a seesaw is not rotating, you know the torque on each side is balanced (Figure 5.26). An object in rotational equilibrium can also be spinning at constant speed, like the blades on a fan.

Using rotational equilibrium

Rotational equilibrium is often used to determine unknown forces. Any object that is not moving must have a net torque of zero *and* a net force of zero. Balances used in schools and scales used in doctors' offices use balanced torques to measure weight. When using a scale, you must slide small masses away from the axis of rotation until the scale balances. Moving the mass increases its lever arm and its torque. Engineers study balanced torques and forces when designing bridges and buildings.

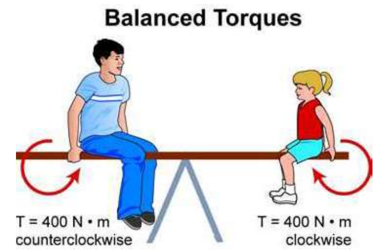
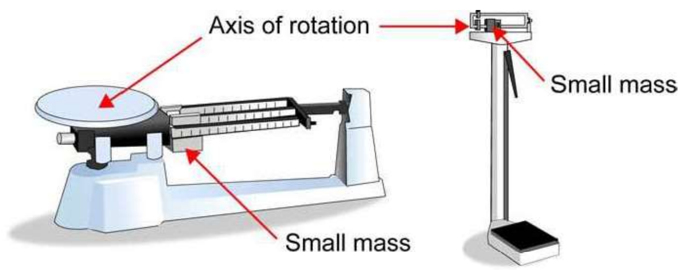


Figure 5.26: *A seesaw is in rotational equilibrium when the two torques are balanced.*



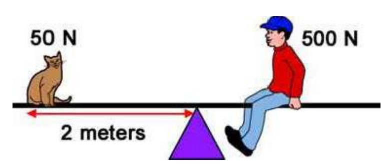


Figure 5.27: *How far must the boy sit from the center of the seesaw in order to balance?*

5.4 Section Review

1. List two ways in which torque is different from force.
2. In what units is torque measured?
3. Explain how the same force can create different amounts of torque on an object.
4. What is the net torque on an object in rotational equilibrium?
5. A boy and a cat sit on a seesaw as shown in Figure 5.27. Use the information in the picture to calculate the torque created by the cat. Then calculate the boy's distance from the center of the seesaw.

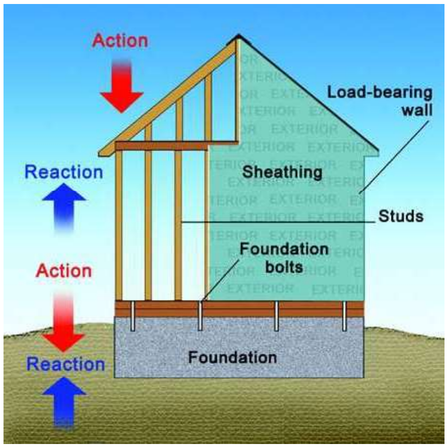
Architecture: Forces in Equilibrium

Even though the builders of the Pyramids of Egypt lived more than 4,000 years ago, they understood how a building needed to be designed to remain standing. They used back-breaking efforts and many attempts to refine their design, keeping ideas that worked and discarding those that didn't. Some pyramids collapsed while they were being built, others lasted decades or centuries before failing, and some are still standing—over 80 pyramids remain in Egypt.

Over time the trial and error process of designing buildings has evolved into a hybrid of science, engineering, and art called architecture. Modern buildings can be very complex and intricate. Yet just as with the most primitive buildings, the structural forces involved must be in equilibrium for the building to stand the test of time. The Pyramids of Giza have lasted about 5,200 years. How do modern buildings compare to the pyramids?

Most buildings today are not pyramids, but are rectangular with four walls and a roof. Also, few buildings are now constructed entirely of limestone and granite like the pyramids. Even though the shape and construction materials are quite different, the ultimate goal is the same—creation of a free-standing structure. A basic box shaped building must have walls that support a roof. But what does "support" mean in terms of forces and equilibrium?

The physics of walls

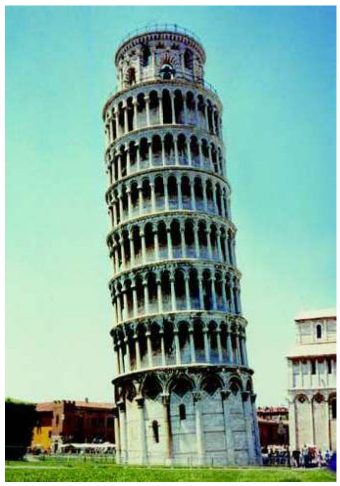


By staying upright, walls provide a platform for a roof. Walls that carry the weight of the roof are called load-bearing walls. Here is where Newton's second law applies—force equals mass times acceleration ($F = ma$). Gravity pulls down on the mass of the roof, creating a force (*weight*). Why doesn't the roof accelerate down toward the ground because of this force? This is where Newton's third law applies—for every action there is an equal and opposite reaction. If the roof isn't moving down, the load bearing walls must be pushing back on the roof in the upwards direction with a force equal to the weight of the roof. This action-reaction pair is in equilibrium, both forces balancing out one another, bringing up Newton's first law. An object at rest remains at rest until acted on by an unbalanced force.

The foundation

The weight force of the walls and roof combined pushes down on the foundation. Just like the action-reaction force pair of the roof and wall, the wall and foundation create an action-reaction pair. Hopefully, this will be in equilibrium too. If the foundation can't provide an equal and opposite reaction, the building will not be in equilibrium. This is the case with the Leaning Tower of Pisa.

The tower began leaning even before construction was finished. The soft soil under the tower began to compress, indicating that the foundation was not large enough to provide the force to equal the weight of the tower. The tower has actually sunk into the ground quite a few feet. One side has compressed more than the other, causing the tower to lean. Towers need to be built with very stable and solid foundations. New York City is an island with a thick layer of extremely stable bedrock just below the topsoil, making it an ideal place for skyscrapers.



The roof

The roof is one of the most important design features of a building. While its major function is to protect the inside of the building from outside elements, it also contributes to the beauty of the structure. The design of the roof therefore must be a balance of form and function. And, it must be able to be supported by the walls and foundation. One of the most famous roofs in the world is the dome that tops the church of Santa Maria del Fiore in Florence, Italy. Called the Duomo, this roof seems to defy the laws of physics.

Fillippo Brunelleschi (1377-1446), an accomplished goldsmith and sculptor, travelled to Rome for a two-year study of ancient roman architecture with fellow artist and friend Donatello. The Pantheon, an amazing dome finished in A.D. 126 was of particular interest to Brunelleschi.

Upon his return to Florence, he finished a design for his dome in 1402, but kept it secret. He claimed to be able to build a self-supporting dome without the use of scaffolding, an outlandish claim many deemed impossible. Yet even without explaining how he would accomplish the feat, construction began.

Brunelleschi knew that large domes tended to sag in the middle, lowering the roof and creating huge forces that pushed outward on the supporting base. He used an ingenious double-walled design, one to be seen from inside the church and another on the outside. He also used intricate herringbone patterns of brickwork and huge timbers linked together with metal fasteners around the dome to balance the forces like hoops on a barrel. This design was so innovative and beautiful it is said to have inspired many of the Renaissance's greatest artists including Leonardo da Vinci and fellow Florentine Michelangelo.

Questions:

1. What are the action-reaction pairs in a typical building?
2. Why is New York City an ideal place for skyscrapers?
3. Explain the elements of Brunelleschi's dome design that keeps it standing.

Table of Contents

Chapter 5 Review

Understanding Vocabulary

Select the correct term to complete the sentences.

scalar	components	friction
magnitude	normal force	lubricant
equilibrium	torque	lever arm
static friction	resultant	vector
Hooke's law	rotational equilibrium	free-body diagram

Section 5.1

1. A ____ has both magnitude and direction.
2. A ____ has magnitude and no direction.
3. ___ is the size or amount of something and includes a unit of measurement.
4. Every vector can be represented as the sum of its ____.
5. A ____ shows all of the forces acting on an object.

Section 5.2

6. If a book sits on a table, the table exerts an upward ____ on the book.
7. The sum of two vectors is called the _____.
8. ____ is the relationship between a spring's change in length and the force it exerts.
9. When the net force acting on an object is zero, the object is in ____.

Section 5.3

10. ____ is a force that resists the motion of objects.
11. The type of friction between objects that are not moving is called ____.
12. A fluid used to reduce friction is called a ____.

Section 5.4

13. ____ is the action that causes objects to rotate.
14. An object is in ____ if the net torque on it is zero.
15. The longer the _____ of a force, the greater the torque.

Reviewing Concepts

Section 5.1

1. Give two examples of vector quantities and two examples of scalar quantities.
2. List the three different ways in which a force vector can be described.
3. Explain how to find the components of a vector.
4. Explain the Pythagorean theorem using an equation and a picture.
5. A 200-newton television sits on a table. Draw a free-body diagram showing the two forces acting on the television.

Section 5.2

6. What is the net force on an object in equilibrium?
7. What is the mathematical meaning of the word *normal*?
8. As you sit on a chair, gravity exerts a downward force on you.
   a. What other force acts on you?
   b. What is the direction of this other force?
   c. What do you know about the magnitude or strength of this other force?
9. If an object is in equilibrium, then the forces in the x-direction must add to _____, and the forces in the y-direction must add to _____.
10. You pull one end of a spring to the right.
    a. What is the action force?
    b. What is the reaction force?
    c. How do the directions of the two forces compare?
    d. How do the strengths of the two forces compare?
11. What happens to a spring's force as you stretch it a greater amount?
12. What do you know about a spring if it has a large spring constant?

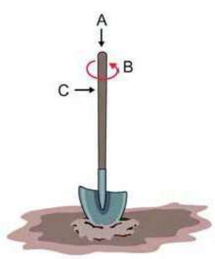
Section 5.3

13. List four types of friction.
14. In which direction does friction act?
15. What is the difference between static friction and sliding friction?

16. What causes friction?
17. Why is it much easier to slide a cardboard box when it is empty compared to when it is full of heavy books?
18. Explain two ways friction can be reduced.
19. Is friction something we always want to reduce? Explain.

Section 5.4

20. How are torque and force similar? How are they different?
21. Which two quantities determine the torque on an object?
22. In what units is torque measured? Do these units have the same meaning as they do when measuring work? Explain.
23. Why is it easier to loosen a bolt with a long-handled wrench than with a short-handled one?
24. In which of the case would a force cause the greatest torque on the shovel? Why?
    a. You press straight down on the shovel so it stays straight up and down.
    b. You twist the shovel like a screwdriver.
    c. You push to the right on the shovel's handle so it tilts toward the ground.



25. What does it mean to say an object is in rotational equilibrium?

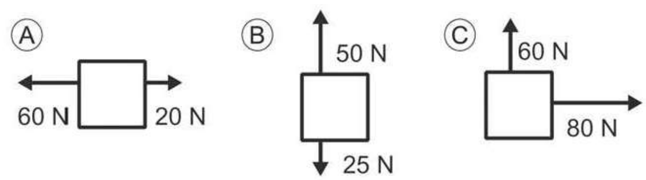
Solving Problems

Section 5.1

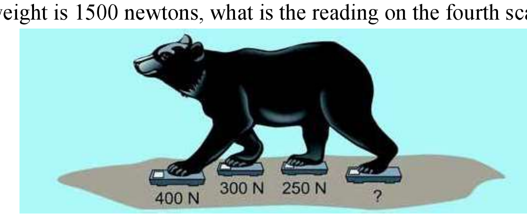
1. Use a ruler to draw each of the following vectors with a scale of 1 centimeter = 1 newton.
    a. (5 N, 0°)
    b. (7 N, 45°)
    c. (3 N, 90°)
    d. (6 N, 30°)
2. Use a ruler to draw each of the following vectors. State the scale you use for each.
    a. (40 N, 0°)
    b. (20 N, 60°)
    c. (100 N, 75°)
    d. (500 N, 90°)
3. Use a scaled drawing to find the components of each of the following vectors.State the scale you use for each.
    a. (5 N, 45°)
    b. (8 N, 30°)
    c. (8 N, 60°)
    d. (100 N, 20°)

Section 5.2

4. Find the net force on each box.



5. A 20-kilogram monkey hangs from a tree limb by both arms. Draw a free-body diagram showing the forces on the monkey. Hint: 20 kg is not a force!
6. You weigh a bear by making him stand on four scales as shown.Draw a free-body diagram showing all the forces acting on the bear. If his weight is 1500 newtons, what is the reading on the fourth scale?



7. A spring has a spring constant of 100 N/m. What force does the spring exert on you if you stretch it a distance of 0.5 meter?

Table of Contents

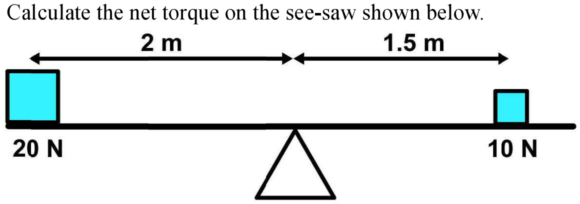
8. If you stretch a spring a distance of 3 cm, it exerts a force of 50 N on your hand. What force will it exert if you stretch it a distance of 6 cm?

Section 5.3

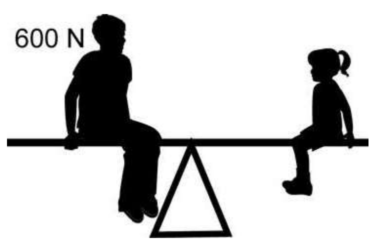
9. Your backpack weighs 50 N. You pull it across a table at a constant speed by exerting a force of 20 N to the right. Draw a free-body diagram showing all *four* forces on the backpack. State the strength of each.
10. You exert a 50 N force to the right on a 300 N box but it does not move. Draw a free-body diagram for the box. Label all the forces and state their strengths.

Section 5.4

11. You push down on a lever with a force of 30 N at a distance of 2 meters from its fulcrum. What is the torque on the lever?
12. You use a wrench to loosen a bolt. It finally turns when you apply 300 N of force at a distance of 0.2 m from the center of the bolt. What torque did you apply?
13. A rusty bolt requires 200 N-m of torque to loosen it. If you can exert a maximum force of 400 N, how long a wrench do you need?
14. Calculate the net torque on the see-saw shown below.



15. You and your little cousin sit on a see-saw. You sit 0.5 m from the fulcrum, and your cousin sits 1.5 m from the fulcrum. You weigh 600 N. How much does she weigh?



Applying Your Knowledge

Section 5.1

1. Is it possible to arrange three forces of 100 N, 200 N, and 300 N so they are in equilibrium? If so, draw a diagram. This is similar to balancing the forces acting on a post.
2. Draw the forces acting on a ladder leaning against a building. Assume you are standing half-way up the ladder. Assume the wall of the building and the ground exert only normal forces on the ladder.

Section 5.2

3. Civil engineers analyze forces in equilibrium when designing bridges. Choose a well-known bridge to research. Some of the questions you might want to answer are listed below.
   a. Who designed the bridge?
   b. How long did it take to build?
   c. Which type of bridge is it?
   d. How much weight was it designed to hold?
   e. What makes this bridge special?

Section 5.3

4. Many cars today have “antilock breaks” that help prevent them from skidding. Research to find out how antilock breaks work.

Section 5.4

5. Can an object be in rotational equilibrium but not have a net force of zero exerted on it? Can an object have a net force of zero but not be in rotational equilibrium? Explain your answers using diagrams.