5 PROJECTILE MOTION

Objectives

- Distinguish between a vector quantity and a scalar quantity. (5.1)
- Explain how to find the resultant of two perpendicular vectors. (5.2)
- Describe how the components of a vector affect each other. (5.3)
- Describe the components of projectile motion. (5.4)
- Describe the downward motion of a horizontally launched projectile. (5.5)
- Describe how far below an imaginary straight-line path a projectile falls. (5.6)

discover!

MATERIALS cardboard, tape, soda straws, pencil, ruler, two marbles

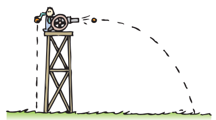
EXPECTED OUTCOME Marble 2 will hit Marble 1.

ANALYZE AND CONCLUDE

1. Yes; the speed of Marble 2
2. Same
3. Gravity accelerates both marbles equally.

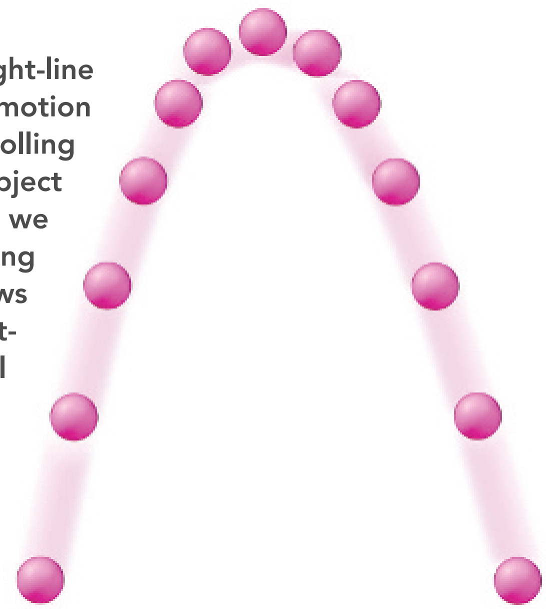
TEACHING TIP Known to many as the "monkey and hunter," this demonstration illustrates that vertical acceleration is independent of initial velocity. Many students find it difficult to believe that if the target is released at the instant the projectile is launched, the way to hit the target is to aim right at it.

5 PROJECTILE MOTION



THE BIG IDEA Projectile motion can be described by the horizontal and vertical components of motion.

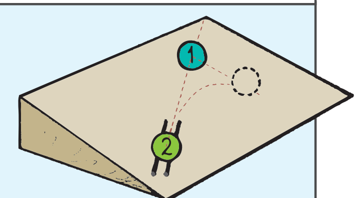
In the previous chapter, we studied simple straight-line motion—linear motion. We distinguished between motion with constant velocity, such as a bowling ball rolling horizontally, and accelerated motion, such as an object falling vertically under the influence of gravity. Now we extend these ideas to nonlinear motion—motion along a curved path. Throw a baseball and the path it follows is a curve. This curve is a combination of constant-velocity horizontal motion and accelerated vertical motion. We'll see that the velocity of a thrown ball at any instant has two independent "components" of motion—what happens horizontally is not affected by what happens vertically.



discover!

How Should You Aim to Hit a Falling Target?

1. On one corner of a rectangular piece of rigid cardboard, tape two 5-cm lengths of soda straws so they form a trough. Angle the straws to point toward the diagonal corner of the cardboard.
2. Draw a straight line passing through the center of the soda straws and extending to the top of the cardboard.
3. Tilt the cardboard so that Marble 1 will roll downhill. Hold Marble 1 in the upper right corner of the cardboard on the line you've drawn. Place a second marble in the trough formed by the two soda straws.
4. At the same time, release Marble 1 and launch Marble 2 by giving it a flick with your finger.



Analyze and Conclude

1. **Observing** Did you hit Marble 1? If so, what determined the point of collision?
2. **Predicting** What would happen if you used marbles with different masses?
3. **Making Generalizations** Why do the two marbles fall the same vertical distance from the line in the same amount of time?

5.1 Vector and Scalar Quantities

It is often said that a picture is worth a thousand words. Sometimes a picture explains a physics concept better than an equation does. Physicists love sketching doodles and equations to explain ideas. Their doodles often include arrows, where each arrow represents the magnitude and the direction of a certain quantity. The quantity might be the tension in a stretched rope, the compressive force in a squeezed spring, or the change in velocity of an airplane flying in the wind.

A quantity that requires both magnitude and direction for a complete description is a vector quantity. Recall from Chapter 4 that velocity differs from speed in that velocity includes direction in its description. Velocity is a vector quantity, as is acceleration. In later chapters we'll see that other quantities, such as momentum, are also vector quantities. For now we'll focus on the vector nature of velocity.

Recall from Chapter 2 that a quantity that is completely described by magnitude only is a scalar quantity. Scalars can be added, subtracted, multiplied, and divided like ordinary numbers. **A vector quantity includes both magnitude and direction, but a scalar quantity includes only magnitude.** When 3 kg of sand is added to 1 kg of cement, the resulting mixture has a mass of 4 kg. When 5 liters of water are poured from a pail that has 8 liters of water in it, the resulting volume is 3 liters. If a scheduled 60-minute trip has a 15-minute delay, the trip takes 75 minutes. In each of these cases, no direction is involved. We see that descriptions such as 10 kilograms north, 5 liters east, or 15 minutes south have no meaning.

CONCEPT CHECK How does a scalar quantity differ from a vector quantity?

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For: Links on vectors
Visit: www.Scilinks.org
Web Code: csn – 0501

Physics on the Job

Air-Traffic Controller

Busy airports have many aircraft landing or taking off every minute. Air-traffic controllers are responsible for guiding pilots to their destinations. Using an understanding of vectors, air-traffic controllers determine the proper speed and direction of an aircraft by taking into account the velocity of the wind, path of the aircraft, and local air traffic. They use radar equipment as well as their view from the control tower to follow the motion of all aircraft flying near the airport.

▶ **Teaching Tip** You may begin your discussion of the Discover! activity by asking students how they would aim if gravity were not acting on either marble. Most will answer directly at the target for, in this case, the target marble would remain stationary and the projectile marble would follow the line drawn on the cardboard. With gravity acting, the answer remains the same. Gravity accelerates both marbles equally causing them to fall the same distance from the line in the same amount of time. A hit is achieved regardless of the mass of the marbles or the initial speed of the projectile.

Unlike the standard monkey and hunter demonstration, the vertical acceleration due to gravity is not equal to $-9.8\ m/s^2$. The use of a slanted surface "dilutes" gravity and hence reduces the acceleration.

Three-ring binders may be used in lieu of a sheet of cardboard. They are perfectly suited for this experiment since they provide an angled surface.

5.1 Vector and Scalar Quantities

CONCEPT CHECK A vector quantity includes both magnitude and direction, but a scalar quantity includes only magnitude.

Teaching Resources

- **Reading and Study Workbook**
- **Presentation*EXPRESS***
- **Interactive Textbook**
- **Conceptual Physics Alive! DVDs *Vectors and Projectiles***

5.2 Velocity Vectors

Common Misconceptions
1 + 1 = 2, always.

FACT Vectors of magnitudes 1 and 1 do not always produce a resultant vector of magnitude 2.

▶ **Teaching Tip** Spend one class period discussing only vector combination, and a different class period discussing vector resolution.

▶ **Teaching Tip** Call attention to the fact that on a windy day one can run faster when running with the wind rather than against the wind. Similarly, a plane is often late or early in arriving at its destination due to wind conditions.

▶ **Teaching Tip** Represent speed with an arrow. Since you are representing magnitude with the length of the arrow, and direction with the arrowhead, you are now talking about velocity—a vector quantity.

▶ **Teaching Tip** Discuss Figure 5.2 and variations of wind conditions that are only with or against the motion of the aircraft. Parallel this with a similar treatment of boats sailing with and against a stream.

Ask **How fast would an airplane move over the ground if it had an airspeed of 100 km/h when flying into a strong gale of 100 km/h?** *It would have a ground speed of zero, just as birds are often seen to have when facing into a strong wind.*

FIGURE 5.1 ▲
This vector, scaled so that 1 cm = 20 km/h, represents 60 km/h to the right.

5.2 Velocity Vectors

The vector in Figure 5.1 is scaled so that 1 centimeter represents 20 kilometers per hour. It is 3 centimeters long and points to the right; therefore it represents a velocity of 60 kilometers per hour to the right, or 60 km/h east.

The velocity of something is often the result of combining two or more other velocities. For example, an airplane's velocity is a combination of the velocity of the airplane relative to the air and the velocity of the air relative to the ground, or the wind velocity. Consider a small airplane slowly flying north at 80 km/h relative to the surrounding air. Suppose there is a tailwind blowing north at a velocity of 20 km/h. This example is represented with vectors in Figure 5.2. Here the velocity vectors are scaled so that 1 cm represents 20 km/h. Thus, the 80-km/h velocity of the airplane is shown by the 4-cm vector, and the 20-km/h tailwind is shown by the 1-cm vector. With or without vectors we can see that the resulting velocity is going to be 100 km/h. Without the tailwind, the airplane travels 80 kilometers in one hour relative to the ground below. With the tailwind, it travels 100 kilometers in one hour relative to the ground below.

Now suppose the airplane makes a U-turn and flies *into* the wind. The velocity vectors are now in opposite directions. The resulting speed of the airplane is 80 km/h − 20 km/h = 60 km/h. Flying against a 20-km/h wind, the airplane travels only 60 kilometers in one hour relative to the ground.

We didn't have to use vectors to answer questions about tailwinds and headwinds, but we'll now see that vectors are useful for combining velocities that are not parallel.

Consider an 80-km/h airplane flying north caught in a strong crosswind of 60 km/h blowing from west to east. Figure 5.3 shows vectors for the airplane velocity and wind velocity. The scale is 1 cm = 20 km/h. The sum of these two vectors, called the *resultant*, is the diagonal of the rectangle described by the two vectors.

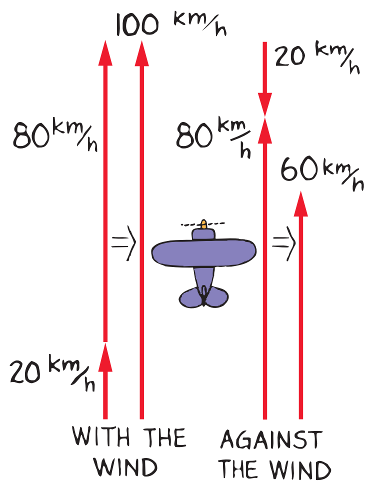


FIGURE 5.2 ▲
The airplane's velocity relative to the ground depends on the airplane's velocity relative to the air and on the wind's velocity.

FIGURE 5.3 ▶
An 80-km/h airplane flying in a 60-km/h crosswind has a resultant speed of 100 km/h relative to the ground.

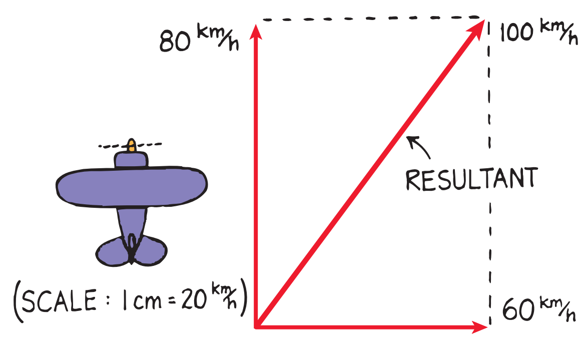
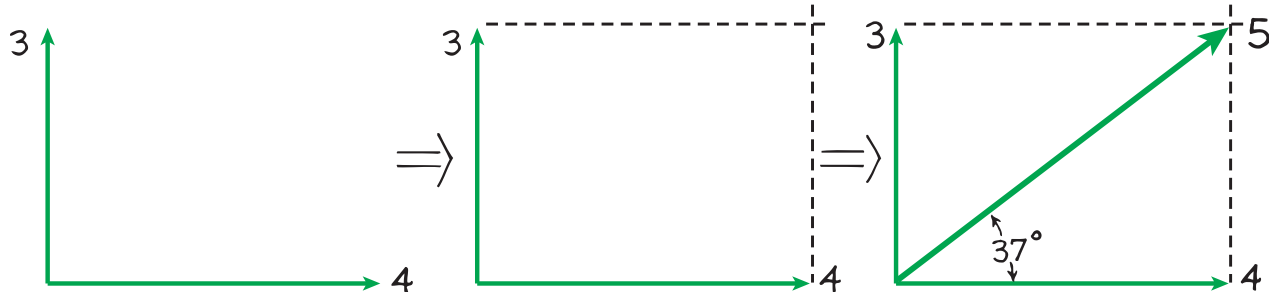
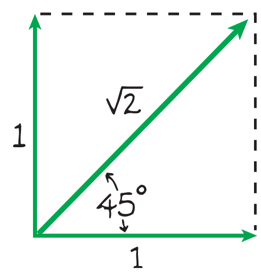


FIGURE 5.4 ▼
The 3-unit and 4-unit vectors add to produce a resultant vector of 5 units, at 37° from the horizontal.



The resultant of two perpendicular vectors is the diagonal of a rectangle constructed with the two vectors as sides. We learned this in Chapter 2. Here, the diagonal of the constructed rectangle measures 5 cm, which represents 100 km/h. So relative to the ground, the airplane moves 100 km/h northeasterly.[5.2.1]

In Figure 5.4 we see a 3-unit vector at right angles to a 4-unit vector. Can you see that they make up the sides of a rectangle, and when added vectorially they produce a resultant of magnitude of 5? (Note that $5^2 = 3^2 + 4^2$.)



◀ **FIGURE 5.5**
The diagonal of a square is $\sqrt{2}$, or 1.414, times the length of one of its sides.

In the special case of adding a pair of equal-magnitude vectors that are at right angles to each other, we construct a square, as shown in Figure 5.5. For any square, the length of its diagonal is $\sqrt{2}$, or 1.414, times either of its sides. Thus, the resultant is $\sqrt{2}$ times either of the vectors. For instance, the resultant of two equal vectors of magnitude 100 acting at right angles to each other is 141.4.[5.2.2]

CONCEPT CHECK **What is the resultant of two perpendicular vectors?**

think!

Suppose that an airplane normally flying at 80 km/h encounters wind at a right angle to its forward motion—a crosswind. Will the airplane fly faster or slower than 80 km/h? *Answer: 5.2*

▶ **Teaching Tip** Tell your students that vector methods, in their modern form, go back only about 100 years. They were introduced by Oliver Heaviside in England and developed by Gibbs in the United States.

▶ **Teaching Tip** Continue with the airplane and wind, and consider a wind at right angles to the nose of the plane, as shown in Figure 5.3. Ask if a crosswind will speed up an airplane, slow it down, or have no effect. Don't be surprised when many in your class say "no effect." Logic leads to this wrong answer. Here is where vectors are needed. At this point introduce the parallelogram rule. Only consider 90° cases, and invoke the Pythagorean theorem—3-4-5 triangles for a start.

▶ **Teaching Tip** After discussing the geometry of the square in Figure 5.5, follow up with a boat sailing across a stream.

Ask **How fast will a boat that normally travels 10 km/h in still water be moving with respect to land if it sails directly across a stream that flows at 10 km/h?** *14.14 km/h*

CONCEPT CHECK The resultant of two perpendicular vectors is the diagonal of a rectangle constructed with the two vectors as sides.

Teaching Resources

- **Reading and Study Workbook**
- **Problem-Solving Exercises in Physics** *3-1*
- **Transparency** *6*
- **Presentation***EXPRESS*
- **Interactive Textbook**
- **Next-Time Question** *5-2*

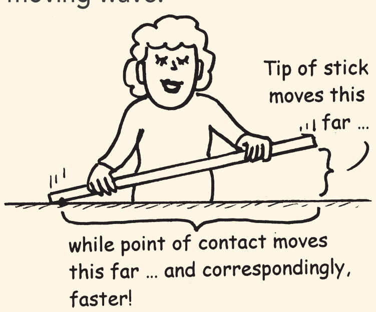
5.3 Components of Vectors

Key Terms
components, resolution

▶ **Teaching Tip** Consider an airplane flying across a wind that does not meet it at 90°. The Pythagorean theorem cannot be used directly. Introduce the technique of parallelogram construction, first for rectangles as in Figure 5.4, and then for vectors.

Physics of Sports

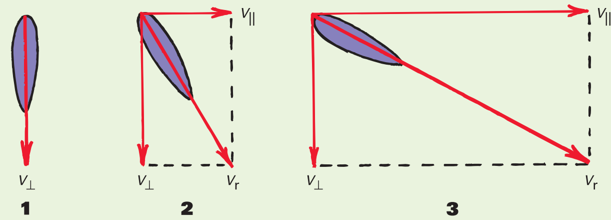
TEACHING TIP Hold a meter stick at an angle of about 30° or so above your table. Slowly lower the stick, maintaining its angle, so that it misses the table's edge. Ask your class to note the point of "contact" of the stick with the table—the point that moves horizontally across the surface as the stick is lowered. Ask for a comparison of the speed of the stick and the speed of the point. They should see the point moves about twice as fast. Tip the stick to about 10° with the horizontal and repeat. The point moves faster. The effect is similar for the surfer who angles across the crest of a moving wave!



CONCEPT CHECK The perpendicular components of a vector are independent of each other.

Physics of Sports

Surfing Surfing nicely illustrates component and resultant vectors. (1) When surfing in the same direction as the wave, our velocity is the same as the wave's velocity, $v_{\perp}$. This velocity is called $v_{\perp}$ because we are moving perpendicular to the wave front. (2) To go faster, we surf at an angle to the wave front. Now we have a component of velocity parallel to the wave front, $v_{\parallel}$, as well as the perpendicular component $v_{\perp}$. We can vary $v_{\parallel}$, but $v_{\perp}$ stays relatively constant as long as we ride the wave. Adding components, we see that when surfing at an angle to the wave front, our resultant velocity, v_r, exceeds $v_{\perp}$. (3) As we increase our angle relative to the wave front, the resultant velocity also increases.



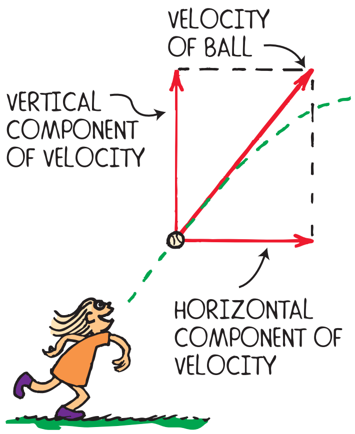


FIGURE 5.6 ▲
A ball's velocity can be resolved into horizontal and vertical components.

5.3 Components of Vectors

Often we will need to change a single vector into an equivalent set of two *component* vectors at right angles to each other. Any vector can be "resolved" into two component vectors at right angles to each other, as shown in Figure 5.6. Two vectors at right angles that add up to a given vector are known as the **components** of the given vector they replace. The process of determining the components of a vector is called **resolution.** Any vector drawn on a piece of paper can be resolved into vertical and horizontal components that are perpendicular. ✓ **The perpendicular components of a vector are independent of each other.**

Vector resolution is illustrated in Figure 5.7. Vector *V* represents a vector quantity. First, vertical and horizontal lines are drawn from the tail of the vector (top). Second, a rectangle is drawn that encloses the vector *V* as its diagonal (bottom). The sides of this rectangle are the desired components, vectors *X* and *Y*.

CONCEPT CHECK How do components of a vector affect each other?

FIGURE 5.7 ▶
Vectors *X* and *Y* are the horizontal and vertical components of a vector *V*.

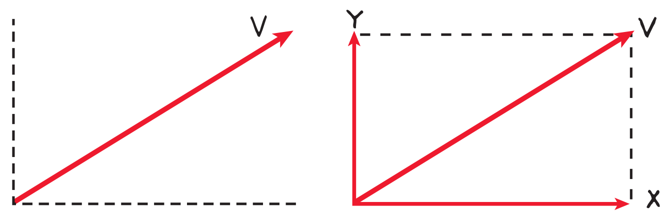
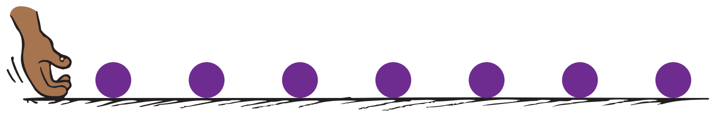
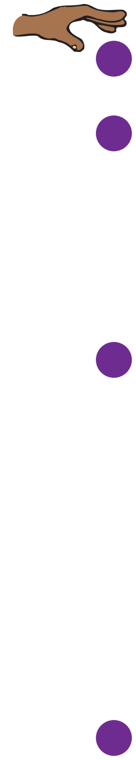


FIGURE 5.8 ▼
Projectile motion can be separated into components.

a. Roll a ball along a horizontal surface, and its velocity is constant because no component of gravitational force acts horizontally.



b. Drop it, and it accelerates downward and covers a greater vertical distance each second.



5.4 Projectile Motion

A cannonball shot from a cannon, a stone thrown into the air, a ball rolling off the edge of a table, a spacecraft circling Earth—all of these are examples of *projectiles*. A **projectile** is any object that moves through the air or space, acted on only by gravity (and air resistance, if any). Projectiles near the surface of Earth follow a curved path that at first seems rather complicated. However, these paths are surprisingly simple when we look at the horizontal and vertical components of motion separately.

✅ **The horizontal component of motion for a projectile is just like the horizontal motion of a ball rolling freely along a level surface without friction.** When friction is negligible, a rolling ball moves at constant velocity. The ball covers equal distances in equal intervals of time as shown in Figure 5.8a. With no horizontal force acting on the ball there is no horizontal acceleration. The same is true for the projectile—when no horizontal force acts on the projectile, the horizontal component of velocity remains constant.

✅ **The vertical component of a projectile's velocity is like the motion for a freely falling object.** Gravity acts vertically downward. Like a ball dropped in midair, a projectile accelerates downward as shown on the right in Figure 5.8b. Its vertical component of velocity changes with time. The increasing speed in the vertical direction causes a greater distance to be covered in each successive equal time interval. Or, if the ball is projected upward, the vertical distances of travel decrease with time on the way up.

Most important, the horizontal component of motion for a projectile is completely independent of the vertical component of motion. Each component is independent of the other. Their combined effects produce the variety of curved paths that projectiles follow.

CONCEPT CHECK **Describe the components of projectile motion.**

5.4 Projectile Motion

Key Term
projectile

Common Misconception
An object at rest will drop to the ground faster than the same object moving horizontally at high speed.

FACT The horizontal component of an object's motion has no effect on the vertical component.

▶ **Teaching Tip** Consider a bowling ball rolling along a bowling alley: Gravity pulls it downward, completely perpendicular to the alley with no horizontal component of force, even if it rolls off the edge of the alley like a ball rolling off a tabletop.

Demonstration

Show the independence of horizontal and vertical motions with a spring-loaded apparatus that will launch a ball horizontally while at the same time dropping another that falls vertically. Students will see (and hear) that they hit the floor at the same time. Then announce, "Gravity does not take a holiday on moving objects."

CONCEPT CHECK The horizontal component of projectile motion is like the motion of a ball rolling freely along a level surface without friction. The vertical component is like the motion of a freely falling object.

Teaching Resources

- **Concept-Development Practice Book** *5-1*
- **Laboratory Manual** *16*
- **Probeware Lab Manual** *3*

5.5 Projectiles Launched Horizontally

▶ **Teaching Tip** Compare the downward motions of each ball, and note they are the same. Investigate the sideways motion of the projected ball and see that it moves equal horizontal distances in equal times. Here we see the law of inertia at play. State that an object does not change its speed in a given direction if there is no force acting in that direction.

▶ **Teaching Tip** Point to some target at the far side of your classroom and ask your class to imagine you are going to shoot a rock to the target with a slingshot. Ask if you should aim directly at the target, above it, or below it. Then ask your class to suppose it takes 1 s for the rock to reach the target. If you aim directly at the target, the rock will fall beneath and miss. How far beneath the target would it strike, if the floor weren't in the way? Encourage class discussion here. When the class agrees it is 5 m, ask how far above should you aim to hit the target. The class should again agree it is 5 m.

CONCEPT CHECK The downward motion of a horizontally launched projectile is the same as that of free fall.

Teaching Resources

- **Next-Time Questions**
  5-1, 5-3

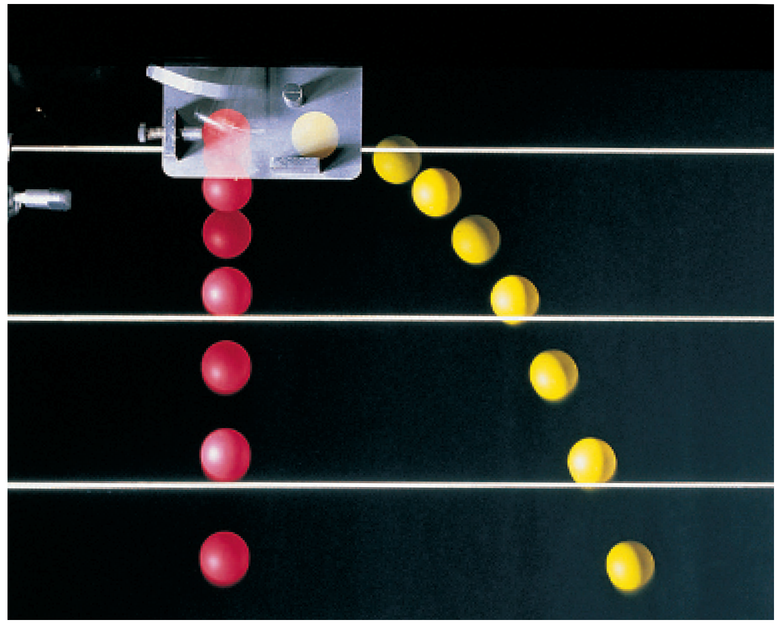


FIGURE 5.9 ▶
A strobe-light photo of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. Notice that in equal times both balls fall the same vertical distance.

The curved path shown in Figure 5.9 is the combination of constant horizontal motion and vertical motion that undergoes acceleration due to gravity.

think!

At the instant a horizontally pointed cannon is fired, a cannonball held at the cannon's side is released and drops to the ground. Which cannonball strikes the ground first, the one fired from the cannon or the one dropped?
Answer: 5.5

5.5 Projectiles Launched Horizontally

Projectile motion is nicely analyzed in the multiple-flash exposure in Figure 5.9. The photo shows equally timed successive positions for two balls. One ball is projected horizontally while the other is simply dropped. Study the photo carefully, for there's a lot of good physics here. Analyze the curved path of the ball by considering the horizontal and vertical velocity components separately. There are two important things to notice. The first is that the ball's horizontal component of motion remains constant. The ball moves the same horizontal distance in the equal time intervals between each flash, because no horizontal component of force is acting on it. Gravity acts only downward, so the only acceleration of the ball is downward. The second thing to note is that both balls fall the same vertical distance in the same time. The vertical distance fallen has nothing to do with the horizontal component of motion. ✓ **The downward motion of a horizontally launched projectile is the same as that of free fall.**

The path traced by a projectile accelerating only in the vertical direction while moving at constant horizontal velocity is a *parabola.* When air resistance is small enough to neglect—usually for slow-moving or very heavy projectiles—the curved paths are parabolic.

Toss a stone from a cliff and its path curves as it accelerates toward the ground below. Figure 5.10a shows how the trajectory is a combination of constant horizontal motion and accelerated vertical motion.

CONCEPT CHECK Describe the downward motion of a horizontally launched projectile.

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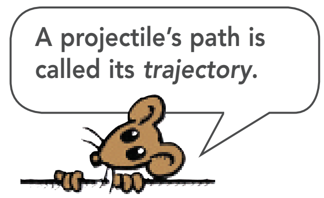
discover!

Which Coin Hits the Ground First?

1. Place a coin at the edge of a table so that it hangs over slightly. Place a second coin on the table some distance from the first coin.
2. Slide the second coin so it hits the first one and both coins fall to the floor below. Which coin hits the ground first?
3. **Think** Does your answer depend on the speed of the coin? Explain.

5.6 Projectiles Launched at an Angle



In Figure 5.10, we see the paths of stones thrown horizontally and at angles upward and downward. The dashed straight lines show the ideal trajectories of the stones if there were no gravity. Notice that the vertical distance that the stone falls beneath the idealized straight-line paths is the same for equal times. This vertical distance is independent of what's happening horizontally.

FIGURE 5.10 ▼
No matter the angle at which a projectile is launched, the vertical distance of fall beneath the idealized straight-line path is the same for equal times.

a. The trajectory of the stone combines horizontal motion with the pull of gravity.

b. The trajectory of the stone combines the upward motion with the pull of gravity.

c. The trajectory of the stone combines downward motion with the pull of gravity.

discover!

MATERIALS coins, table

EXPECTED OUTCOME Students should see (and hear) that both coins hit the floor at the same time.

THINK The horizontal speed of the sliding coin has no effect on the outcome.

5.6 Projectiles Launched at an Angle

Ask How long does the "force of the thrower" act on a ball tossed into the air? *The force exists only while the hand and ball are in contact. Once the ball is released, that propelling force is no more.*

Common Misconception
At the top of its trajectory, the velocity of a projectile is always momentarily zero.

FACT The horizontal velocity of a projectile does not change, so if a projectile has horizontal velocity it will have the same horizontal velocity at the top of its trajectory.

Ask How is the horizontal component of motion affected by the vertical component of motion? *It isn't! The horizontal and vertical components of motion are independent of each other.*

Teaching Tip Discuss Figure 5.11. Call attention to the vertical distances fallen and Table 4.3 on page 56 (the same physics of free fall, only "stretched out horizontally").

Ask **If the cannon were aimed downward instead of upward in Figure 5.11, how would the distances below the new "dashed line" compare?** *The projectile displacements below the dashed line would be no different; 5 m at the end of the first second, 20 m at the end of the second second, and so on.*

discover!

MATERIALS ruler, string, five beads

EXPECTED OUTCOME The beads hang along the path of a projectile launched at the angle at which the ruler is held.

THINK The beads hang at distances proportional to the square of the distance out on the ruler.

think!

A projectile is launched at an angle into the air. Neglecting air resistance, what is its vertical acceleration? Its horizontal acceleration?
Answer: 5.6.1

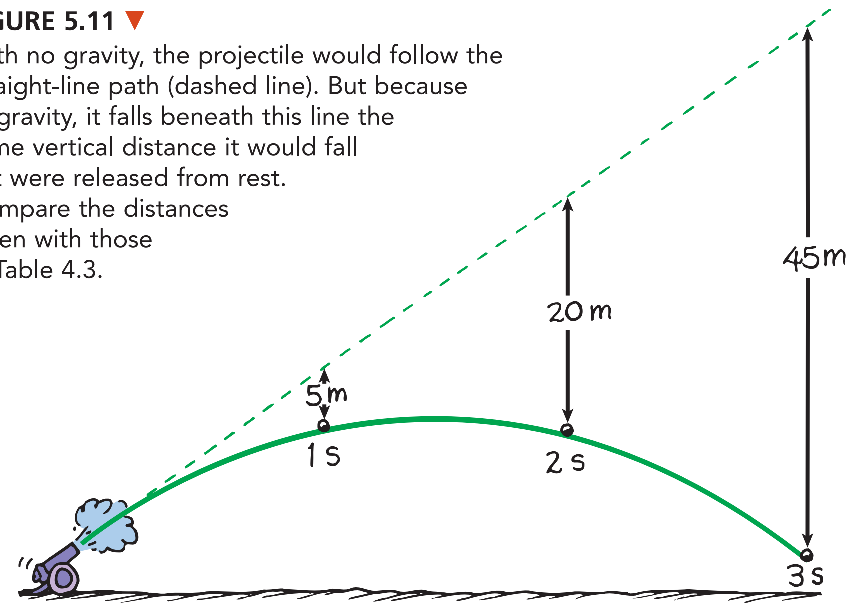
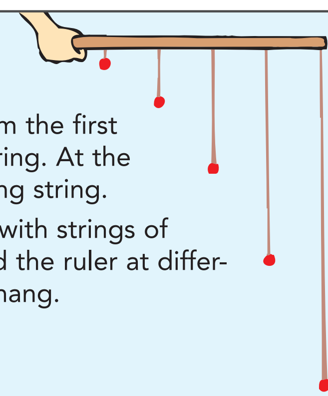


FIGURE 5.11 With no gravity, the projectile would follow the straight-line path (dashed line). But because of gravity, it falls beneath this line the same vertical distance it would fall if it were released from rest. Compare the distances fallen with those in Table 4.3.

Figure 5.11 shows specific vertical distances for a cannonball shot at an upward angle. If there were no gravity, the cannonball would follow the straight-line path shown by the dashed line. But there is gravity, so this doesn't occur. What happens is that the cannonball continuously falls beneath the imaginary line until it finally strikes the ground. The vertical distance it falls *beneath any point on the dashed line* is the same vertical distance it would fall if it were dropped from rest and had been falling for the same amount of time. This distance, introduced in Chapter 4, is given by $d = \frac{1}{2}gt^2$, where t is the elapsed time. Using the value of 10 m/s^2 for g in the equation yields $d = 5t^2$ meters.

discover!

How Can You Model Projectile Motion?



1. Mark a ruler at five equal spaces. From the first mark, hang a bead on a 1-cm long string. At the next mark, hang a bead on a 4-cm long string.
2. Hang beads on the next three marks with strings of lengths 9 cm, 16 cm, and 25 cm. Hold the ruler at different angles and see where the beads hang.
3. **Think** Why is this model accurate?

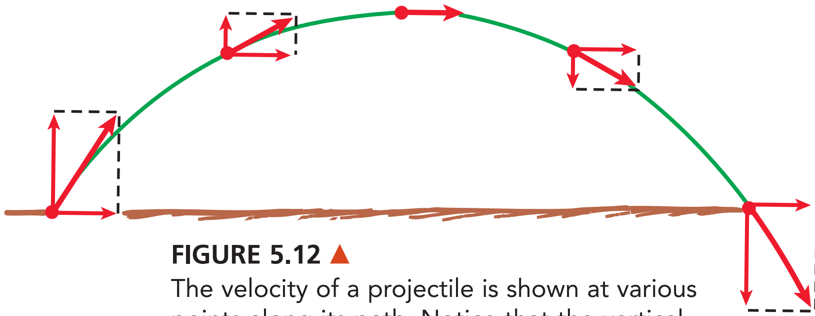


FIGURE 5.12 ▲
The velocity of a projectile is shown at various points along its path. Notice that the vertical component changes while the horizontal component does not. Air resistance is neglected.

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Height We can put this another way. Toss a projectile skyward at some angle and pretend there is no gravity. After so many seconds *t*, it should be at a certain point along a straight-line path. But due to gravity, it isn't. Where is it? The answer is, it's directly below that point. How far below? The answer is $5t^2$ meters below that point. How about that? **The vertical distance a projectile falls below an imaginary straight-line path increases continually with time and is equal to $5t^2$ meters.**

Note also from Figure 5.11 that since there is no horizontal acceleration, the cannonball moves equal horizontal distances in equal time intervals. That's because there is no horizontal acceleration. The only acceleration is vertical, in the direction of Earth's gravity.

Figure 5.12 shows vectors representing both the horizontal and vertical components of velocity for a projectile on a parabolic path. Notice that the horizontal component is always the same and that only the vertical component changes. Note also that the actual resultant velocity vector is represented by the diagonal of the rectangle formed by the vector components. At the top of the path the vertical component shrinks to zero, so the velocity there *is* the same as the horizontal component of velocity at all other points. Everywhere else the magnitude of velocity is greater, just as the diagonal of a rectangle is greater than either of its sides.

Range Figure 5.13 shows the path traced by a projectile with the same launching speed but at a steeper angle. Notice that the initial velocity vector has a greater vertical component than when the projection angle is less. This greater component results in a higher path. However, since the horizontal component is less, the range is less.

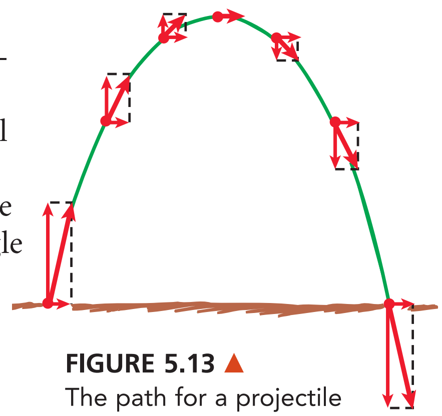

FIGURE 5.13 ▲
The path for a projectile fired at a steep angle. Again, air resistance is neglected.

▶ **Teaching Tip** Discuss the relative vectors in Figure 5.12. Note that the horizontal component doesn't change (because no horizontal force acts). Note that the vertical component does change (going upward against gravity, then downward with gravity). Point out that the same is true of the steeper angle of Figure 5.13.

Ask **True or False? The velocity of a projectile at its highest point is zero.** *False; the vertical component of velocity, not the velocity itself, is zero at the highest point, unless the projectile moves straight upward.* **What can be said of the velocity of the projectile at its highest point?** *At its highest point and neglecting air resistance, the velocity of a projectile will be the same as its horizontal component of velocity at any other point.*

The equal ranges for projectiles launched at complementary angles is quite interesting. The explanation requires a knowledge of sine and cosine trig functions, so let that wait until a future physics course (save and direct your students' energy to more fertile ground).

PAUL

Interestingly, the maximum height of a projectile following a parabolic path is nicely given by sketching an isosceles triangle with the base equal to the range of the projectile. Let the two side angles be equal to the launch angle. The maximum height reached by the projectile is equal to half the altitude of the triangle.

PAUL

Ask **To direct water to flowers that are farthest away, at what angle should a water hose be held?** *Ideally 45°, and somewhat less if you're holding it high off the ground*

Physics of Sports

Hang Time Revisited

TEACHING TIP Ask if one could jump higher when on a moving skateboard or in a moving bus. It should be clear that the answer is no. But one can usually jump higher from a running jump. It is a mistake to assume that the horizontal motion is responsible for the higher jump and longer hang time. The action of running likely produces a greater force between the foot and the ground, which gives a greater vertical liftoff component of velocity. This greater force against the floor, and not any reduction of effect of gravity on a horizontally moving body, is the explanation. Stress that the vertical component of velocity alone determines vertical height and hang time.

FIGURE 5.14 ▶
The paths of projectiles launched at the same speed but at different angles. The paths neglect air resistance.

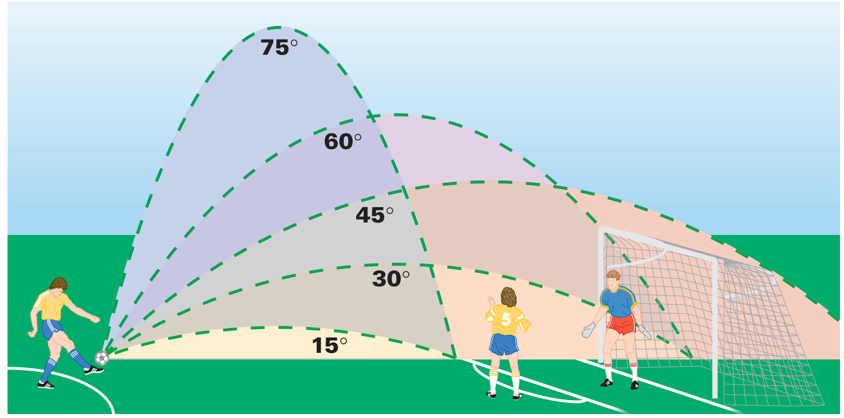

FIGURE 5.15 ▲
Maximum range is attained when the ball is batted at an angle of nearly 45°.

Horizontal Ranges Figure 5.14 shows the paths of several projectiles all having the same initial speed but different projection angles. The figure neglects the effects of air resistance, so the paths are all parabolas. Notice that these projectiles reach different heights (altitude) above the ground. They also travel different horizontal distances, that is, they have different *horizontal ranges.*

The remarkable thing to note from Figure 5.14 is that the same range is obtained for two different projection angles—angles that add up to 90 degrees! For example, an object thrown into the air at an angle of 60 degrees will have the same range as if it were thrown at 30 degrees with the same speed. Of course, for the smaller angle the object remains in the air for a shorter time. Maximum range is usually attained at an angle of 45°. For a thrown javelin, on the other hand, maximum range is achieved for an angle quite a bit less than 45°, because the force of gravity on the relatively heavy javelin is significant during launch. Just as you can't throw a heavy rock as fast upward as sideways, so it is that the javelin's launch speed is reduced when thrown upward.

Physics of Sports

Hang Time Revisited

Recall our discussion of hang time in Chapter 4. We stated that the time one is airborne during a jump is independent of horizontal speed. Now we see why this is so—horizontal and vertical components of motion are independent of each other. The rules of projectile motion apply to jumping. Once the feet are off the ground, if we neglect air resistance, the only force acting on the jumper is gravity. Hang time depends only on the vertical component of liftoff velocity. It turns out that jumping force can be somewhat increased by the action of running, so hang time for a running jump usually exceeds that for a standing jump. However, once the feet are off the ground, only the vertical component of liftoff velocity determines hang time.

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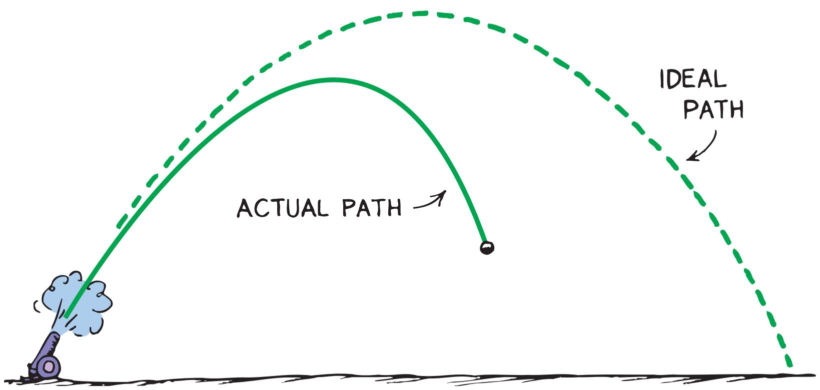


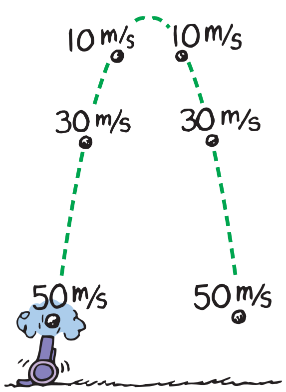
◀ **FIGURE 5.16**
In the presence of air resistance, the path of a high-speed projectile falls below the idealized parabola and follows the solid curve.

Speed We have emphasized the special case of projectile motion for negligible air resistance. As we can see in Figure 5.16, when the effect of air resistance is significant, the range of a projectile is diminished and the path is not a true parabola.

If air resistance is negligible, a projectile will rise to its maximum height in the same time it takes to fall from that height to the ground. This is due to the constant effect of gravity. The deceleration due to gravity going up is the same as the acceleration due to gravity coming down. The speed it loses going up is therefore the same as the speed it gains coming down, as shown in Figure 5.17. So the projectile hits the ground with the same speed it had originally when it was projected upward from the ground.

For short-range projectile motion such as a batted ball in a baseball game, we usually assume the ground is flat. However, for very long range projectiles the curvature of Earth's surface must be taken into account. We'll see that if an object is projected fast enough, it will fall all the way around Earth and become an Earth satellite! More about satellites in Chapter 14.

CONCEPT CHECK **Describe how far below an imaginary straight-line path a projectile falls.**

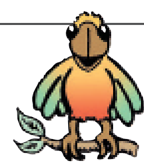


◀ **FIGURE 5.17**
Without air resistance, the speed lost while the cannonball is going up equals the speed gained while it is coming down. The time to go up equals the time to come down.

think!

At what point in its path does a projectile have minimum speed?
Answer: 5.6.2

The longest time a jumper is airborne for a standing jump (hang time) is 1 second, for a record 1.25 meters (4 ft) height. Can anyone in your school jump that high, raising their center of gravity 1.25 meters above the ground? Not likely!



▶ **Teaching Tip** Acknowledge the large effect that air resistance (drag) has on the foregoing analysis, particularly for fast-moving objects such as cannonballs. A batted baseball, for example, travels only about 60% as far in air as it would in a vacuum. Its curved path is no longer a parabola, as Figure 5.16 indicates.

CONCEPT CHECK The vertical distance a projectile falls below an imaginary straight-line path increases continually with time and is equal to $5t^2$ meters.

Teaching Resources

- **Reading and Study Workbook**
- **Concept-Development Practice Book** *5-2, 5-3*
- **Problem-Solving Exercises in Physics** *3-2*
- **Laboratory Manual** *17*
- **Transparencies** *7, 8*
- **Presentation*EXPRESS***
- **Interactive Textbook**

5 REVIEW

Teaching Resources

- Teacher*EXPRESS*
- Virtual Physics Lab *7*
- Conceptual Physics Alive! DVDs *Vectors and Projectiles*

5 REVIEW

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Concept Summary

- A vector quantity includes both magnitude and direction, but a scalar quantity includes only magnitude.
- The resultant of two perpendicular vectors is the diagonal of a rectangle constructed with the two vectors as sides.
- The perpendicular components of a vector are independent of each other.
- The horizontal component of motion for a projectile is just like the horizontal motion of a ball rolling freely along a level surface without friction. The vertical component of a projectile's velocity is like the motion for a freely falling object.
- The downward motion of a horizontally launched projectile is the same as that of free fall.
- The vertical distance a projectile falls below an imaginary straight-line path increases continually with time and is equal to $5t^2$ meters.

Key Terms

components *(p. 72)*
resolution *(p. 72)*
projectile *(p. 73)*

think! Answers

5.2 A crosswind would increase the speed of the airplane and blow it off course by a predictable amount.

5.5 Both cannonballs fall the same vertical distance with the same acceleration *g* and therefore strike the ground at the same time. Do you see that this is consistent with our analysis of Figure 5.9? Ask which cannonball strikes the ground first when the cannon is pointed at an upward angle. In this case, the cannonball that is simply dropped hits the ground first. Now consider the case when the cannon is pointed downward. The fired cannonball hits first. So upward, the dropped cannonball hits first; downward, the fired cannonball hits first. There must be some angle where both hit at the same time. Do you see it would be when the cannon is pointing neither upward nor downward, that is, when it is pointing horizontally?

5.6.1 Its vertical acceleration is *g* because the force of gravity is downward. Its horizontal acceleration is zero because no horizontal force acts on it.

5.6.2 The minimum speed of a projectile occurs at the top of its path. If it is launched vertically, its speed at the top is zero. If it is projected at an angle, the vertical component of velocity is still zero at the top, leaving only the horizontal component. So the speed at the top is equal to the horizontal component of the projectile's velocity at any point. How about that?

5 ASSESS

Check Concepts

Section 5.1

1. How does a vector quantity differ from a scalar quantity?

2. Why is speed classified as a scalar quantity and velocity classified as a vector quantity?

Section 5.2

3. If a vector that is 1 cm long represents a velocity of 10 km/h, what velocity does a vector 2 cm long drawn to the same scale represent?

4. When a rectangle is constructed in order to add perpendicular velocities, what part of the rectangle represents the resultant vector?

Section 5.3

5. Will a vector at 45° to the horizontal be larger or smaller than its horizontal and vertical components? By how much?

Section 5.4

6. Why does a bowling ball move without acceleration when it rolls along a bowling alley?

7. In the absence of air resistance, why does the horizontal component of velocity for a projectile remain constant while the vertical component changes?

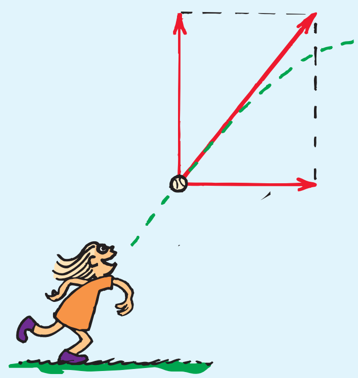
8. How does the downward component of the motion of a projectile compare with the motion of free fall?

Section 5.5

9. At the instant a ball is thrown horizontally over a level range, a ball held at the side of the first is released and drops to the ground. If air resistance is neglected, which ball strikes the ground first?

Section 5.6

10. **a.** How far below an initial straight-line path will a projectile fall in one second?
    b. Does your answer depend on the angle of launch or on the initial speed of the projectile? Defend your answer.



11. Neglecting air resistance, if you throw a ball straight up with a speed of 20 m/s, how fast will it be moving when you catch it?

12. **a.** Neglecting air resistance, if you throw a baseball at 20 m/s to your friend who is on first base, will the catching speed be greater than, equal to, or less than 20 m/s?
    b. Does the speed change if air resistance is a factor?

13. What do we call a projectile that continually "falls" around Earth?

5 ASSESS

Check Concepts

1. Only a vector quantity is specified by direction.
2. Speed has no particular direction; velocity specifies direction.
3. 20 km/h
4. The diagonal
5. Larger; the vector will be $\sqrt{2}$ (or 1.414) times greater than either of the components.
6. There is no net force on the ball.
7. There is no horizontal component of gravitational force.
8. Same; both are under the influence of gravity.
9. They hit the ground at the same time.
10. **a.** 5 m
    b. No; vertical displacement below any line is $1/2gt^2$.
11. 20 m/s
12. **a.** Equal to
    b. Yes, it decreases.
13. An Earth satellite

Think and Rank

14. **a.** B, C, A, D
b. B, D, A, C

15. **a.** C, B, A
b. A, C, B

16. **a.** A = B = C = D
b. A = B = C = D
c. A = C, B = D

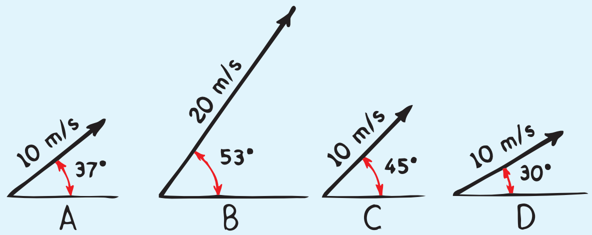
17. D, B, A, C

5 ASSESS *(continued)*

Think and Rank

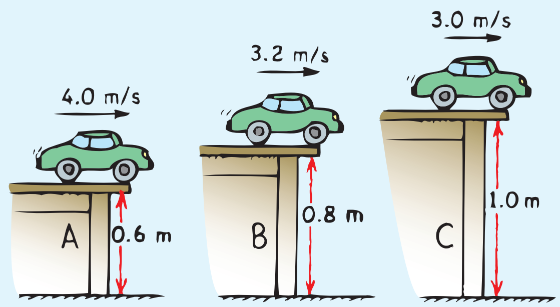
Rank each of the following sets of scenarios in order of the quantity or property involved. List them from left to right. If scenarios have equal rankings, then separate them with an equal sign. (e.g., A = B)

14. The vectors represent initial velocities of projectiles launched at ground level.



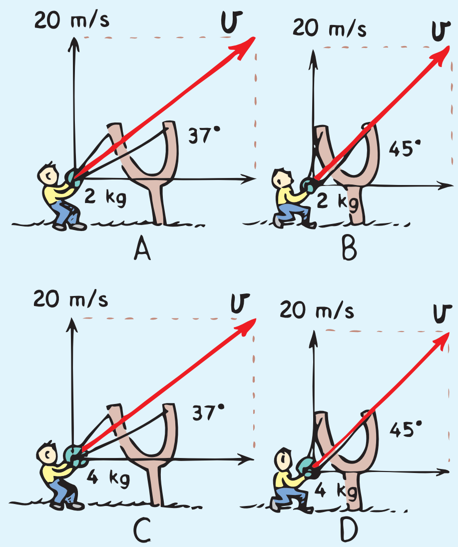
a. Rank them by their vertical components of velocity from greatest to least.
b. Rank them by their horizontal components of velocity from greatest to least.

15. A toy car rolls off tables of various heights at different speeds as shown.



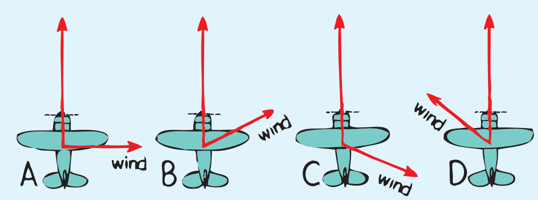
a. Rank them for the time in the air, from greatest to least.
b. Rank them for horizontal range, from greatest to least.

16. Water balloons of different masses are launched by slingshots at different launching velocities *v*. All have the same vertical component of launching velocities.



a. Rank by the *time* in the air, from longest to shortest.
b. Rank by the maximum *height* reached, from highest to lowest.
c. Rank by the maximum *range*, from greatest to least.

17. The airplane is blown off course by wind in the directions shown. Use the parallelogram rule and rank from highest to lowest the resulting speed across the ground.



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Plug and Chug

For Questions 18–19, recall that when two vectors in the same or exactly opposite directions are added, the magnitude of their resultant is the sum or difference of their original magnitudes.

18. Calculate the resultant velocity of an airplane that normally flies at 200 km/h if it encounters a 50-km/h tailwind. If it encounters a 50-km/h headwind.

19. Calculate the magnitude of the resultant of a pair of 100-km/h velocity vectors that are at right angles to each other.

For questions 20–21, recall that the resultant V *of two vectors* A *and* B *at right angles to each other is found using the Pythagorean theorem:*

$$V = \sqrt{A^2 + B^2}$$

20. Calculate the resulting speed of an airplane with an airspeed of 120 km/h pointing due north when it encounters a wind of 90 km/h directed from the west. (Recall, speed is the magnitude of velocity.)

21. Calculate the speed of raindrops hitting your face when they fall vertically at 3 m/s while you're running horizontally at 4 m/s.

Think and Explain

22. Whenever you add 3 and 4, the result is 7. This is true if the quantities being added are scalar quantities. If 3 and 4 are the magnitudes of vector quantities, when will the magnitude of their sum be 5?

23. Christopher can paddle a canoe in still water at 8 km/h. How successful will he be at canoeing upstream in a river that flows at 8 km/h?

24. How does the vertical distance a projectile falls below an otherwise straight-line path compare with the vertical distance it would fall from rest in the same time?

25. The speed of falling rain is the same 10 m above ground as it is just before it hits the ground. What does this tell you about whether or not the rain encounters air resistance?

26. Marshall says that when a pair of vectors are at right angles to each other, the magnitude of their resultant is greater than the magnitude of either vector alone. Renee says he is speaking in generalities and that what he says isn't always true. With whom do you agree?

27. How is the horizontal component of velocity for a projectile affected by the vertical component?

28. Rain falling vertically will make vertical streaks on a car's side window. However, if the car is moving, the streaks are slanted. If the streaks from a vertically falling rain make 45° streaks, how fast is the car moving compared with the speed of the falling rain?

29. An airplane encounters a wind that blows in a perpendicular direction to the direction its nose is pointing. Does the effect of this wind increase or decrease speed across the ground below? Or does it have no effect on ground speed?

30. A projectile is launched vertically at 50 m/s. If air resistance can be neglected, at what speed will it return to its initial level? Where in its trajectory will it have minimum speed?

31. A batted baseball follows a parabolic path on a day when the sun is directly overhead. How does the speed of the ball's shadow across the field compare with the ball's horizontal component of velocity?

32. When air resistance acts on a projectile, does it affect the horizontal component of velocity, the vertical component of velocity, or both? Defend your answer.

Plug and Chug

18. 200 km/h + 50 km/h = 250 km/h; 200 km/h − 50 km/h = 150 km/h

19. $\sqrt{(100 \text{ km/h})^2 + (100 \text{ km/h})^2}$ = 141 km/h at 45° to either vector.

20. Speed $= v = \sqrt{v_N^2 + v_W^2}$
$= \sqrt{(120 \text{ km/h})^2 + (90 \text{ km/h})^2}$
= 150 km/h

21. Speed $= v = \sqrt{v_x^2 + v_y^2}$
$= \sqrt{(4 \text{ m/s})^2 + (3 \text{ m/s})^2}$
= 5 m/s

Think and Explain

22. They'll add to magnitude 5 when at right angles to each other.

23. Not very, for his speed will be zero relative to the land

24. Same

25. No acceleration; air resistance balances the weight of the raindrops and the raindrops have reached terminal speed.

26. Agree with Marshall, and cite how the diagonal of a rectangle has greater magnitude than either of its sides.

27. Not at all; horizontal and vertical components of velocity are independent of each other.

28. The car moves as fast horizontally as the raindrops fall vertically.

29. Ground speed is increased, in accord with vector rules.

30. 50 m/s; minimum speed at top

31. The same

32. Both components are reduced. Air resistance acts opposite velocity, whether vertical or horizontal.

33. The component along the direction of the road decreases so the car ahead gains distance on you.

34. 15°

35. It depends only on the vertical component of velocity as your feet leave the ground. Once off the ground, the only acceleration (neglecting any effects of air drag) is g, which is vertical. Your vertical liftoff velocity divided by g will be the time you move upward. Double this and you have your hang time.

36. Since in this case the height is the same either way, hang time is the same either way.

Think and Solve

37. **a.** Speed in headwind = 10 m/s − 2 m/s = 8 m/s

b. Speed in tailwind = 10 m/s + 2 m/s = 12 m/s

c. Speed in right-angle crosswind;

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(10\text{ m/s})^2 + (10\text{ m/s})^2}$$
$$= 14\text{ m/s}$$

(45° from his original course)

38. **a.** $v = \sqrt{8^2 + 6^2} = 10$ km/h

b. 10 km/h at 37° upstream (or any speed > 6 km/h with a 6-km/h upstream component)

39. 10 units each

40. $x = v_x t = 30$ m

41. **a.** Vertical component = gt = $(10\text{ m/s}^2)(2\text{ s}) = 20$ m/s
Horizontal component is 15 m/s.

b. $v = \sqrt{v_x^2 + v_y^2}$
$$= \sqrt{(15\text{ m/s})^2 + (20\text{ m/s})^2}$$
$$= 25\text{ m/s}$$

5 ASSESS (continued)

33. You're driving behind a car and wish to pass, so you turn to the left and pull into the passing lane without changing speed. Why does the distance increase between you and the car you're following?

34. Brandon launches a projectile at an angle of 75° above the horizontal, which strikes the ground a certain distance down range. For what other angle of launch at the same speed would the projectile land just as far away?

35. When you jump up, your hang time is the time your feet are off the ground. Does hang time depend on your vertical component of velocity when you jump, your horizontal component of velocity, or both? Defend your answer.

36. The hang time of a basketball player who jumps a vertical distance of 2 feet (0.6 m) is 2/3 second. What will be the hang time if the player reaches the same height while jumping a horizontal distance of 4 feet (1.2 m)?

Think and Solve

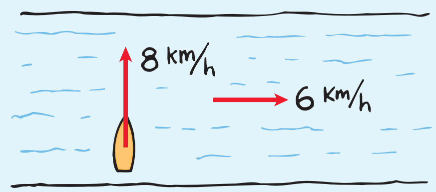
37. Sneezlee flies at a speed of 10 m/s in still air.

a. If he flies into a 2-m/s headwind, how fast will he be traveling relative to the ground below?

b. Relative to the ground below, how fast will he travel when he experiences a 2-m/s tailwind?

c. While flying at 10 m/s, suppose that he encounters a 10-m/s cross wind (coming at a right angle to his heading). What is his speed relative to the ground below?

38. A boat is rowed at 8 km/h directly across a river that flows at 6 km/h, as shown in the figure.



a. What is the resultant speed of the boat?

b. How fast and in what direction can the boat be rowed to reach a destination directly across the river?

39. If a 14-unit vector makes an angle of 45° with the horizontal, what are its horizontal and vertical components?

40. Harry accidentally falls out of a helicopter that is traveling at 15 m/s. He plunges into a swimming pool 2 seconds later. Assuming no air resistance, what was the horizontal distance between Harry and the swimming pool when he fell from the helicopter?

41. Refer to the previous problem.

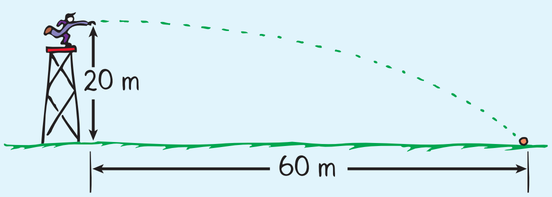
a. What are the horizontal and vertical components of Harry's velocity just as he hits the water?

b. Show that Harry hits the water at a speed of 25 m/s.

42. Harry and Angela look from their balcony to a swimming pool below that is 15 m from the bottom of their building. They estimate the balcony is 45 m high and wonder how fast they would have to jump horizontally to succeed in reaching the pool. What is your answer?

43. A girl throws a slingshot pellet directly at a target that is far enough away to take one-half second to reach. How far below the target does the pellet hit? How high above the target should she aim?

44. The boy on the tower in the figure below throws a ball a distance of 60 m, as shown. At what speed, in m/s, is the ball thrown?



45. A cannonball launched with an initial velocity of 141 m/s at an angle of 45° follows a parabolic path and hits a balloon at the top of its trajectory. Neglecting air resistance, how fast is it going when it hits the balloon? What is the acceleration of the cannonball just before it hits the balloon?

46. Joshua throws a stone horizontally from a cliff at a speed of 20 m/s, which strikes the ground 2.0 seconds later.

a. Use your knowledge of vectors and show that the stone strikes the ground at a speed of about 28 m/s.

b. At what angle does the ball strike the ground?

47. On a bowling alley, Isabella rolls a bowling ball that covers a distance of 10 meters in 1 second. The speed of the ball is 10 m/s. If the ball were instead dropped from rest off the edge of a building, what would be its speed at the end of 1 second?

48. A bowling ball is moving at 10 m/s when it rolls off the edge of a tall building. What is the ball's speed one second later? (*Hint:* think vectors!)

49. Calculate Hotshot Harry's hang time if he moves horizontally 3 m during a 1.25-m high jump. What is his hang time if he moves 6 m horizontally during this jump?

50. Megan rolls a ball across a lab bench y meters high and the ball rolls off the edge of the bench with horizontal speed v.

a. From the equation $y = \frac{1}{2}gt^2$, which gives the vertical distance y an object falls from rest, derive an equation that shows the time t taken for the ball to reach the floor.

b. Write an equation showing how far the ball will land from a point on the floor directly below the edge of the bench.

c. Calculate the time in the air and the landing location for v = 1.5 m/s and a bench height of 1.2 m.

More Problem-Solving Practice
Appendix F

42. $v_x = x/t$ = 15 m/3 s = 5 m/s (because t = 3 s for a 45-m vertical fall)

43. $y = 1/2gt^2$ = 1/2(10 m/s^2) × (0.5 s)^2 = 1.25 m; 1.25 m

44. 30 m/s (ball takes 2 s to fall 20 m)

45. The horizontal component at the top is 100 m/s (the side of a square whose diagonal is 141). The acceleration at the top is g.

46. a. 2 s later horizontal component of v is still 20 m/s, and vertical v component = gt = 10 m/s^2 × 2 s = 20 m/s. Then,

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(20\text{ m/s})^2 + (20\text{ m/s})^2}$$
$$= 28\text{ m/s}$$

b. At 2 s, horizontal and vertical velocities are the same, a right triangle formed with 45° angle

47. From rest, vertical speed = gt = (10 m/s^2)(1 s) = 10 m/s.

48. After 1 s, ball's horizontal component of v is still 10 m/s, and vertical component is also 10 m/s. Then,

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(10\text{ m/s})^2 + (10\text{ m/s})^2}$$
$$= 14\text{ m/s}$$

(at 45° with the horizontal)

49. 1 s, both cases

50. a. Call downward positive. Then from $y = 1/2gt^2$,

$t^2 = 2y/g$

$t = \sqrt{(2y/g)}$

b. $x = vt = v\sqrt{(2y/g)}$

c. $t = \sqrt{(2y/g)}$ = $\sqrt{2(1.2\text{ m})(10\text{ m/s}^2)}$ = 0.49 s; $x = vt$ = (1.5 m/s)(0.49 s) = 0.74 m

Teaching Resources

- Computer Test Bank
- Chapter and Unit Tests