14 SATELLITE MOTION

Objectives

- Explain how the speed of a satellite in circular orbit around Earth is related to the distance an object falls in the first second due to gravity. (14.1)
- Describe the motion of a satellite in a circular orbit. (14.2)
- Describe the shape of the path of a satellite in an orbit around Earth. (14.3)
- Apply the energy conservation law to describe changes in the PE and KE of a satellite in different portions of an elliptical orbit. (14.4)
- State Kepler's three laws of planetary motion. (14.5)
- Determine the vertical speed required to ensure a projectile can "escape" Earth. (14.6)

discover!

MATERIALS a mass, a 1-m-long string, a support

EXPECTED OUTCOME When the mass is tapped sideways, its motion describes an ellipse. This is the same oval-shaped path traced out by a satellite.

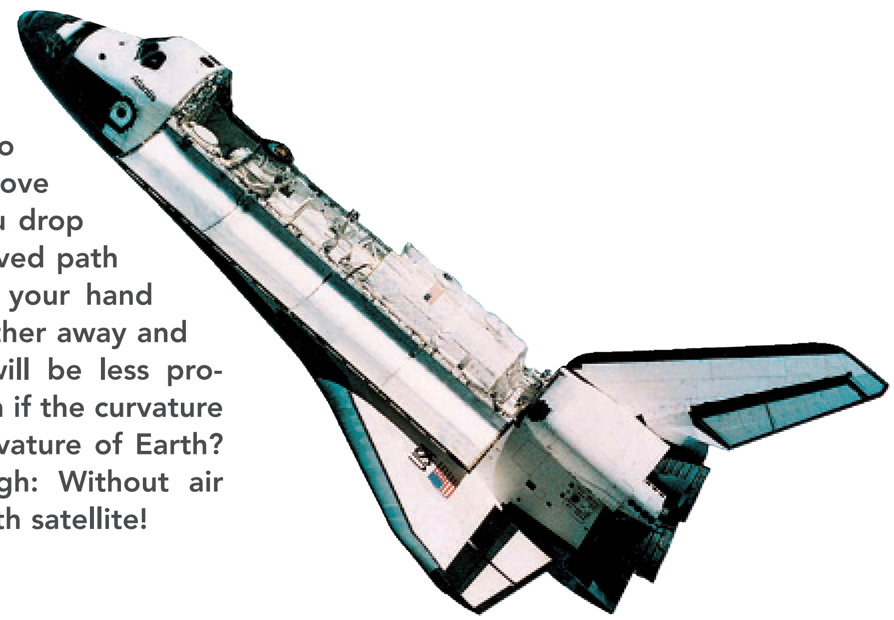
ANALYZE AND CONCLUDE

1. The mass traces out a straight line.
2. With a greater force, the mass traces out a wider path.
3. The mass should not affect the path of the pendulum if the initial conditions are the same.

14 SATELLITE MOTION

THE BIG IDEA The path of an Earth satellite follows the curvature of the Earth.

If you drop a stone, it will fall in a straight-line path to the ground below. If you move your hand horizontally as you drop the stone, it will follow a curved path to the ground. If you move your hand faster, the stone will land farther away and the curvature of the path will be less pronounced. What would happen if the curvature of the path matched the curvature of Earth? The answer is simple enough: Without air resistance, you'd have an Earth satellite!



discover!

What Happens When You Disturb the Path of a Pendulum?

1. Make a pendulum from a mass and a 1-m long string. Tie the free end of the string to a support.
2. Set the pendulum swinging. It should move back and forth only and not side-to-side.
3. While the pendulum is swinging back and forth, tap the mass sideways.
4. Repeat Step 3 several times, each time tapping the mass with a different force.

Analyze and Conclude

1. **Observing** Describe the original shape of the path of the mass.
2. **Drawing Conclusions** What effect does tapping the mass with different forces have on the shape of the path?
3. **Predicting** How might changing the amount of mass affect the nature of the path?

FIGURE 14.1
The greater the stone's horizontal motion when released, the wider the arc of its curved path.

14.1 Earth Satellites

Simply put, an Earth **satellite** is a projectile moving fast enough to fall continually *around* Earth rather than *into* it. Imagine yourself on a planet that is smaller than Earth as shown in Figure 14.2. Because of the planet's small size and low mass, you would not have to throw the stone very fast to make its curved path match the surface curvature of the planet. If you threw the stone just right, it would follow a circular orbit.

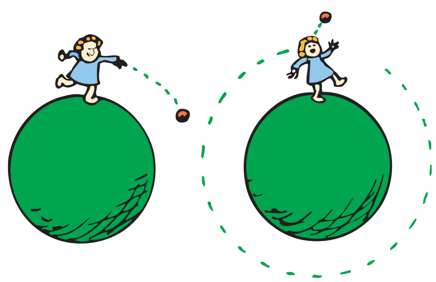


FIGURE 14.2
If you toss the stone horizontally with the proper speed, its path will match the surface curvature of the small planet.

How fast would the stone have to be thrown horizontally for it to orbit Earth? The answer depends on the rate at which the stone falls and the rate at which Earth curves. Recall from Chapter 4 that a stone dropped from rest accelerates downward (or toward the center of Earth) at 10 m/s^2 and falls a vertical distance of 5 meters during the first second. Also recall from Chapter 5 that the same is true of any projectile as it starts to fall. Recall that in the first second a projectile will fall a vertical distance of 5 meters below the straight-line path it would have taken without gravity as shown in Figure 14.3. (It may be helpful to refresh your memory and review Figure 5.11.)

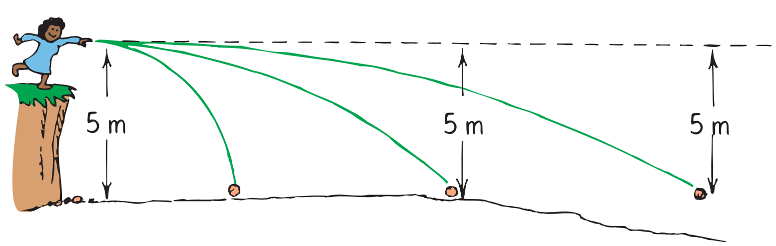


FIGURE 14.3
Throw a stone at any speed and one second later it will have fallen 5 m below where it would have been without gravity.

14.1 Earth Satellites

Key Term
satellite

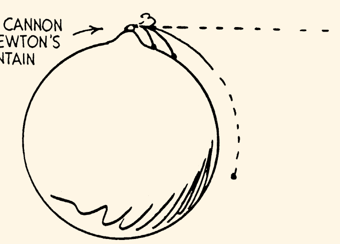
Common Misconception
Satellites are beyond the main pull of a planet's gravitational field.

FACT Gravity keeps a satellite in orbit. Without gravity, the satellite would keep moving in a straight line.

▶ **Teaching Tip** Explain that satellite motion is an extension of projectile motion. Refer back to the idea of the "falling moon" introduced in Chapter 13. Distinguish between falling *around* and falling *into.*

▶ **Teaching Tip** Ask why satellites in close orbit around the moon would have a smaller speed than ones around Earth.

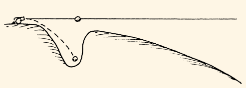
▶ **Teaching Tip** Draw a world on the board. Include a hypothetical mountain at the top. Explain that the mountain is high enough to poke through the atmosphere so that cannonballs fired from it encounter no air resistance. Show how successively greater speeds result in a circular orbit. State that Isaac Newton thought of this idea and calculated the speed of a cannonball necessary for circular orbit.



▶ **Teaching Tip** Sketch Figure 14.4 on the board. Draw a laser on a 1-m-high tripod aimed over a perfectly level desert floor. The beam is straight, but the desert floor curves 4.9 m over an 8000-m (8 km) tangent. (Point out that the sketch is not to scale.) Now replace the laser with a super cannon positioned so it aims along the laser line. Ask your class how far along the laser line a cannonball would go if fired at 2 km/s with no gravity and no air resistance. They should see that it will travel 2 km.

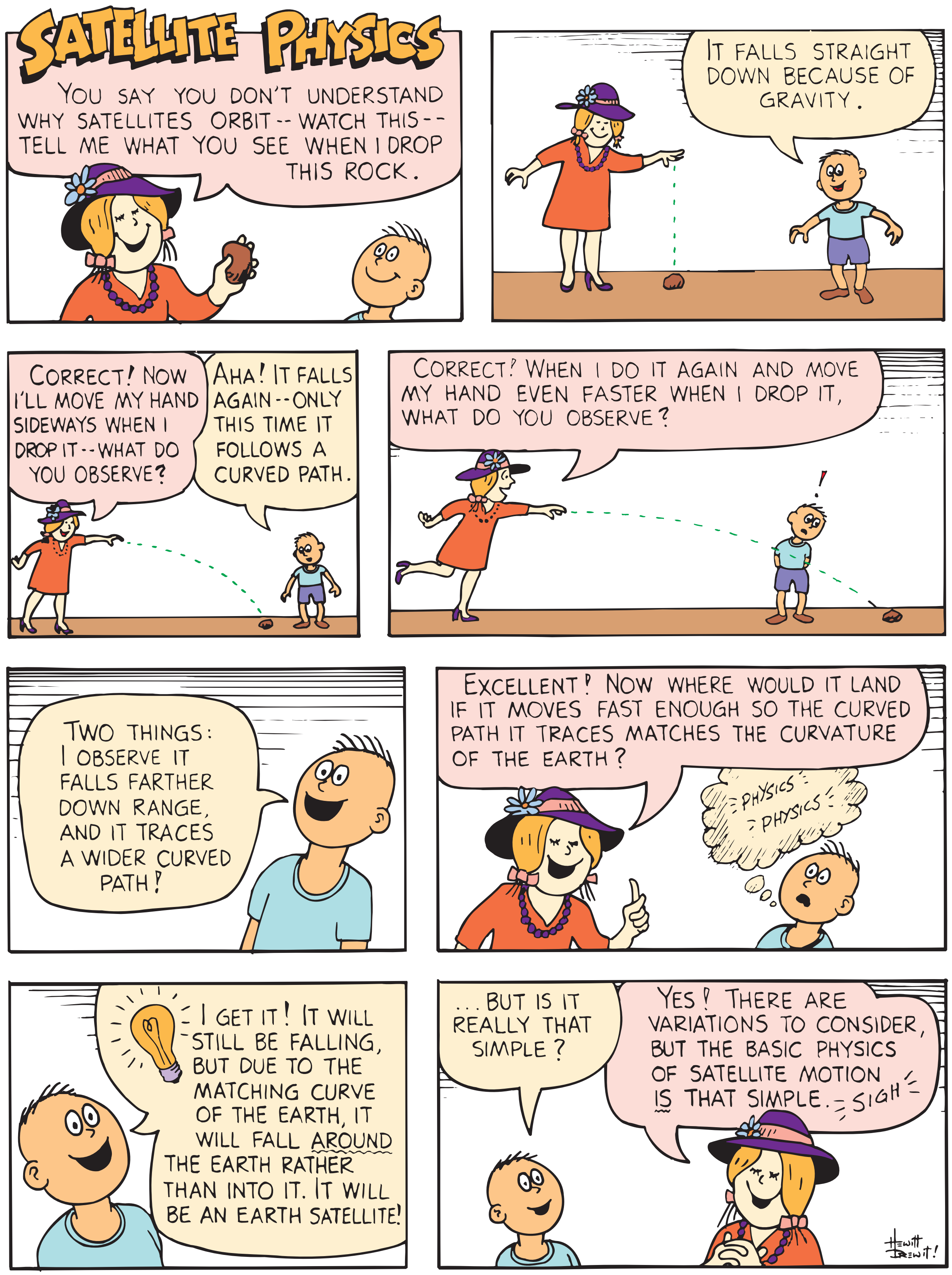
Ask **Is this fast enough for Earth orbit in the presence of Earth's gravity, with no air resistance?** *No. Its speed is insufficient for Earth orbit, because it will crash into the ground before it can reach the 2-km point.* **How far vertically would the cannonball fall beneath the laser line if the ground weren't in the way?** *4.9 m*

▶ **Teaching Tip** Revise the sketch to show the case described above. (You must dig a hole if the cannonball is not to strike Earth's surface.)



▶ **Teaching Tip** Now consider a cannonball fired at 4 km/s. Without gravity or air resistance, it will travel 4 km down the laser path in 1 s.

Ask **Is this fast enough for Earth orbit?** *No, because it will hit the ground before 1 s is up. Emphasize the cannonball will fall a vertical distance of 4.9 m in 1 s whatever its horizontal speed.*



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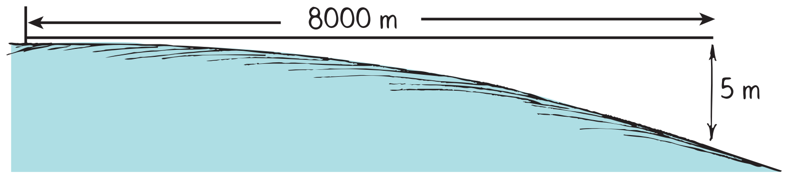
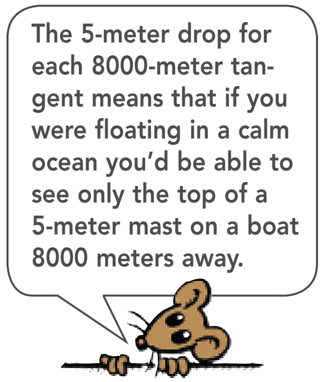


FIGURE 14.4
In the curvature of Earth, the surface drops a vertical distance of nearly 5 meters for every 8000 meters tangent to its surface.

A geometric fact about the curvature of our Earth is that its surface drops a vertical distance of nearly 5 meters for every 8000 meters tangent to its surface as shown in Figure 14.4.

A stone thrown fast enough to go a horizontal distance of 8000 meters during the time (1 second) it takes to fall 5 meters, will orbit Earth. Isn't this speed simply 8000 meters per second? So we see that the orbital speed for close orbit about Earth is 8000 m/s (or 8 km/s). If this doesn't seem to be very fast, convert it to kilometers per hour; you'll see it is an impressive 29,000 km/h (or 18,000 mi/h). At that speed, atmospheric friction would burn an object to a crisp. That's why a satellite must stay about 150 kilometers or more above Earth's surface—to keep from burning due to the friction of the atmosphere.



CONCEPT CHECK Near the surface of Earth, how fast does a stone have to be thrown to orbit Earth?

14.2 Circular Orbits

Interestingly, in circular orbit the speed of a circling satellite is not changed by gravity. We can understand this by comparing a satellite in circular orbit to a bowling ball rolling along a bowling alley as shown in Figure 14.5. Why doesn't the gravity that acts on the bowling ball change its speed? The answer is that gravity is pulling neither forward nor backward—it pulls straight downward, perpendicular to the ball's motion. The bowling ball has no component of gravitational force along the direction of the alley.

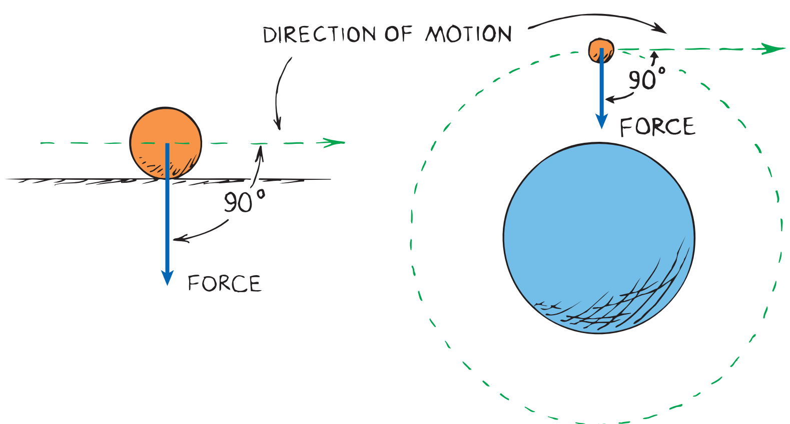
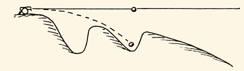
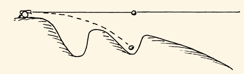


FIGURE 14.5
The speeds of the bowling ball and the satellite are not affected by the force of gravity because there is no horizontal component of the gravitational force.

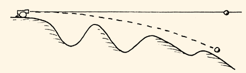
Teaching Tip Dig another hole so your sketch looks like this:



Continue by considering a velocity great enough that the cannonball travels 6 km in 1s. Ask if this is fast enough not to hit the ground (or equivalently, if it is fast enough for Earth orbit!). Repeat the previous line of reasoning, again having to dig a hole, and your sketch looks like this:



Finally, consider a velocity of 8 km/s. Ask if you'll have to dig a hole at this speed. After a pause, ask the class what speed the cannonball must have to orbit Earth. You have now led your class into a "derivation" of orbital speed about Earth.



CONCEPT CHECK A stone thrown fast enough to go a horizontal distance of 8,000 meters during the time (1 second) it takes to fall 5 meters will orbit Earth.

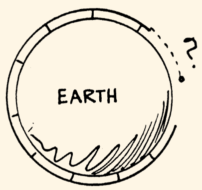
Teaching Resources

- **Reading and Study Workbook**
- **Transparency** *23*
- **Next-Time Question** *14-1*
- **Conceptual Physics Alive! DVDs** *Satellite Motion*
- **Presentation*EXPRESS***
- **Interactive Textbook**

14.2 Circular Orbits

Key Term
period

Teaching Tip Explain that gravity does not change the tangential speed of a satellite; there is no tangential component of gravitation. Consider the effect of gravity on a bowling ball rolling along a level bowling alley. The pull is down, perpendicular to the alley and perpendicular to the direction of motion. Therefore gravity does no work on the ball (Figure 14.5). Consider a bowling alley that completely encircles Earth—elevated so it is above air resistance. The ball would roll indefinitely, always "level."



Ask **How fast would the bowling ball have to be moving for it to clear a broken span and continue moving along the alley on the other side?** *8 km/s. In fact, you could remove the whole alley!* **Would a cannonball fired upward at 8 km/s go into Earth orbit?** *No, it would simply act as a projectile and crash back into Earth at 8 km/s. To circle Earth it must have a tangential speed of 8 km/s.*

Teaching Tip Tell students that the orbit of a satellite with a period of 24 hours is called a *geostationary* or *geosynchronous* orbit.

CONCEPT CHECK A satellite in circular orbit around Earth is always moving perpendicular to gravity and parallel to Earth's surface at constant speed.

Teaching Resources

- **Presentation*EXPRESS***
- **Interactive Textbook**

think!

Satellites in close circular orbit fall about 5 m during each second of orbit. How can this be if the satellite does not get closer to Earth?
Answer: 14.2

The same is true for a satellite in circular orbit. Here a satellite is always moving at a right angle (perpendicular) to the force of gravity. It doesn't move in the direction of gravity, which would increase its speed, nor does it move in a direction against gravity, which would decrease its speed. Instead, the satellite exactly "criss-crosses" gravity, so that no change in speed occurs—only a change in direction. **A satellite in circular orbit around Earth is always moving perpendicular to gravity and parallel to Earth's surface at constant speed.**

For a satellite close to Earth, the time for a complete orbit around Earth, its **period,** is about 90 minutes. For higher altitudes, the orbital speed is less and the period is longer. Communications satellites are located in orbit 6.5 Earth radii from Earth's center, so that their period is 24 hours. This period matches Earth's daily rotation. They are launched to orbit in the plane of Earth's equator, so they are always above the same place on the equator. The moon is farther away, and has a 27.3-day period. The higher the orbit of a satellite, the slower its speed and the longer its period.[14.2]

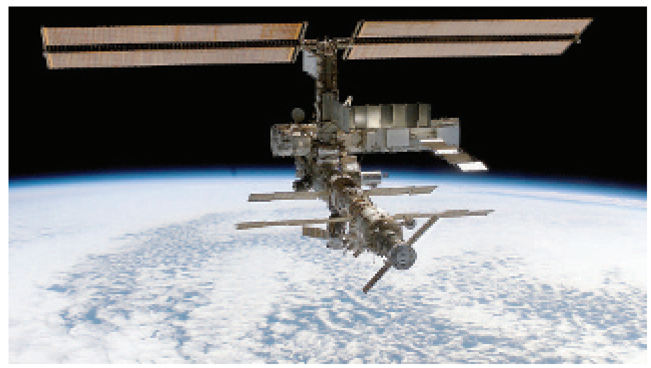


FIGURE 14.6 The ISS and its inhabitants circle 360 km above the Earth, well above its atmosphere, in a state of continual free fall.

The international space station (ISS), shown in Figure 14.6, orbits at 360 kilometers above Earth's surface. Like all satellites, tangential velocity assures that it falls around Earth rather than into it. Acceleration toward Earth is somewhat less than 1 g because of altitude. This acceleration, however, is not sensed by the astronauts; relative to the station, they experience zero g. Over extended periods of time this causes loss of muscle strength and other detrimental changes in the body. Future space travelers, however, can avoid this when space stations rotate (recall our discussion of rotating space habitats in Chapter 12). Rotation effectively supplies a support force and can nicely provide Earth-normal weight.

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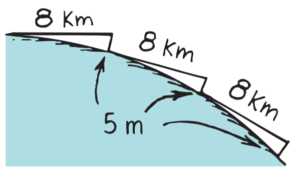
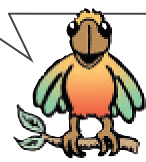


◀ FIGURE 14.7
A satellite in circular orbit close to Earth moves tangentially at 8 km/s. Each second, it falls 5 m beneath each successive 8-km tangent.

Recall from Chapter 13 that Isaac Newton understood satellite motion from his investigation of the moon's motion. He thought about the launching of artificial satellites, for he reasoned that without air resistance, a cannonball could circle Earth and coast indefinitely if it had sufficient speed. As Figure 14.7 demonstrates, he calculated this speed to be the same as 8 km/s. Since such speed was impossible then, he was not optimistic about people launching satellites. What Newton did not consider was multistage rockets—the idea of rockets carried piggyback style on other rockets to reach orbital speed by a succession of rocket firings.

CONCEPT CHECK **Describe the motion of a satellite in relation to Earth's surface and gravity.**



14.3 Elliptical Orbits

✓ **A satellite in orbit around Earth traces an oval-shaped path called an ellipse.** An **ellipse** is the closed path taken by a point that moves in such a way that the sum of its distances from two fixed points is constant. The two fixed points in an ellipse are called **foci.** For a satellite orbiting a planet, the center of the planet is at one focus and the other focus could be inside or outside the planet. An ellipse can be easily constructed by using a pair of tacks, one at each focus, a loop of string, and a pencil, as shown in Figure 14.8. The closer the tacks, the closer the ellipse is to a circle. When the foci are together, the ellipse *is* a circle. A circle is a special case of an ellipse.

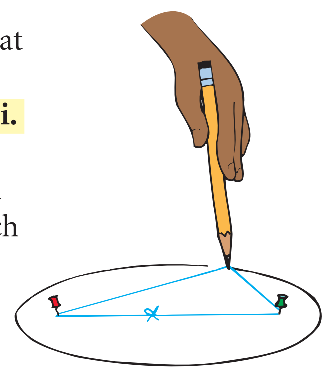


FIGURE 14.8 ▲
A simple method of constructing an ellipse is shown here.

discover!

How Do You Draw Different Ellipses?

1. Draw an ellipse with a loop of string, two tacks, and a pen or pencil, as shown in Figure 14.8.
2. Try different tack spacings for a variety of ellipses. Or draw an ellipse by tracing the edge of the shadow cast by a circular disk on a flat surface.
3. **Think** How can you move the disk to get different ellipses?

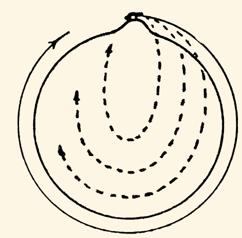
14.3 Elliptical Orbits

Key Terms
ellipse, focus (*plural* foci)

Demonstration

With a loop of string and a pair of small suction cups stuck to the chalkboard, trace an ellipse (as is done in Figure 14.8).

▶ **Teaching Tip** Point out that when you toss a baseball into the air you say its path is parabolic, but strictly speaking, it is a segment of an ellipse. Earth's center is at the far focus of this ellipse. If nothing were in the way, the baseball would follow an eccentric elliptical path and return to its starting point! Earth's center is at the near focus for satellites that trace external elliptical paths around Earth. In this case, nothing occupies the other focal point.



discover!

MATERIALS two tacks, a pen or pencil, a loop of string

EXPECTED OUTCOME Students should produce various ellipses using the method illustrated in Figure 14.8.

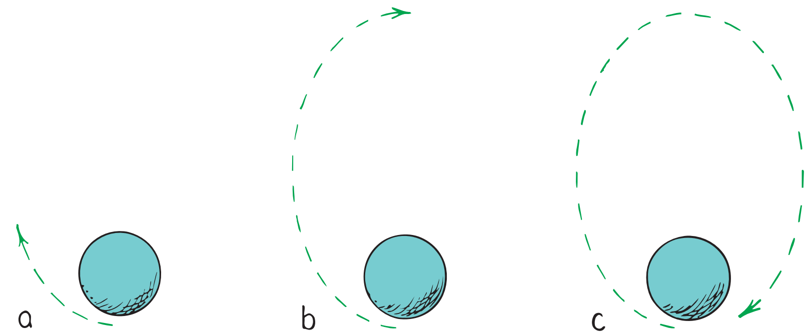
THINK Using the disk method, different ellipses can be produced by varying the tilt of the disk.

▶ **Teaching Tip** Return to the idea of Newton's mountain and consider greater cannonball speeds, starting with say 9 km/s. Show how this speed causes the cannonball to overshoot the path it would take for circular orbit.

Ask **Will the 9 km/s value increase, decrease, or remain the same on the first part of its outward trip?** *Since it is going against gravity, the cannonball will slow to a speed less than its initial speed—a quite different situation than when in circular orbit.*

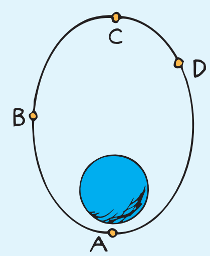
▶ **Teaching Tip** Trace a full ellipse. As you retrace the elliptical path, show with a sweeping motion of your arm how the satellite slows as it recedes from Earth, moving slowest at its farthermost point; then how it speeds up as it falls toward Earth, whipping around Earth at its closest point. The cycle repeats. Point out that this is similar to how a stone thrown upward at an angle slows on the way up, and speeds up on the way down. Planets orbiting about the sun behave the same way. Kepler didn't understand the slowness of planets when farthest from the sun because he did not view them as bodies freely falling around the sun.

FIGURE 14.9 ▶
A satellite moves in an elliptical orbit. **a.** When the satellite exceeds 8 km/s, it overshoots a circle. **b.** At its maximum separation, it starts to come back toward Earth. **c.** The cycle repeats itself.



think!

The orbit of a satellite is shown in the sketch. In which of the positions *A* through *D* does the satellite have the greatest speed? The least speed?
Answer: 14.3

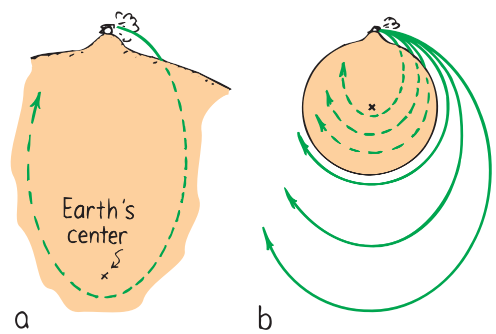


Satellite speed, which is constant in a circular orbit, *varies* in an elliptical orbit. When the initial speed is more than 8 km/s, the satellite overshoots a circular path and moves away from Earth, against the force of gravity. It therefore loses speed. Like a rock thrown into the air, the satellite slows to a point where it no longer recedes, and begins falling back toward Earth. The speed lost in receding is regained as it falls back. The satellite then rejoins its path with the same speed it had initially. The procedure repeats over and over, and an ellipse is traced each cycle as shown in Figure 14.9.

Quite interestingly, the apparently parabolic paths of projectiles such as cannonballs are actually tiny segments of a thin ellipse that extends within and just beyond the center of Earth. Figure 14.10 shows that for speeds less than orbital speeds, the center of Earth is the far focus of the elliptical path. In this case the near focus is close to the launching site and varies for different speeds. When the projectile traces a circular orbit, both foci are together at Earth's center. For elliptical orbits, the near focus is Earth's center and the location of the far focus varies for different speeds.

CONCEPT CHECK **What is the shape of the path of a satellite in an orbit around Earth?**

FIGURE 14.10 ▶
The parabolic paths of projectiles are actually segments of ellipses. **a.** For relatively low speeds, the center of Earth is the far focus. **b.** For greater speeds, the near focus is Earth's center.



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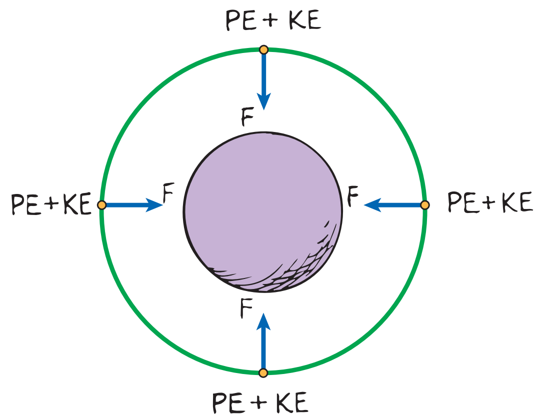


FIGURE 14.11
For a satellite in circular orbit, no component of force acts along the direction of motion. The speed, and thus the KE, cannot change.

14.4 Energy Conservation and Satellite Motion

Recall from Chapter 9 that moving objects have kinetic energy (KE). An object above Earth's surface has potential energy (PE) due to its position. Everywhere in its orbit, a satellite has both KE and PE. **The sum of the KE and PE of a satellite is constant at all points along an orbit.**

In a circular orbit, the distance between a planet's center and the satellite's center is constant, as shown in Figure 14.11. This means that the PE of the satellite is the same everywhere in orbit. So, by the law of conservation of energy, the KE is also constant. Thus, the speed is constant in any circular orbit.

In an elliptical orbit the situation is different. Both speed and distance vary. The **apogee** is the point in a satellite's orbit farthest from the center of Earth. The **perigee** is the point in a satellite's orbit closest to the center of Earth. The PE is greatest when the satellite is at the apogee and least when the satellite is at the perigee. Correspondingly, the KE will be least when the PE is most; and the KE will be most when the PE is least, as Figure 14.12 shows. At every point in the orbit, the sum of the KE and PE is constant.

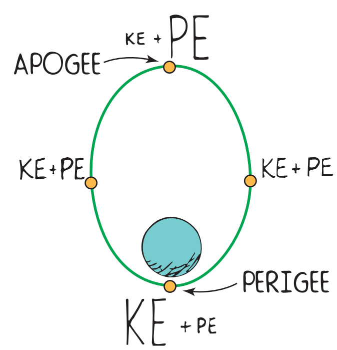


FIGURE 14.12
The sum of KE and PE for a satellite is a constant at all points along an elliptical orbit.

Physics on the Job

Satellite Design Engineer Satellites play an important role in conducting scientific research, obtaining environmental data, and providing communications services. Satellite design engineers are employed by the United States government through NASA and by commercial communications companies. The goal of a satellite design engineer is to design satellites that will orbit at specific distances from Earth, carry the necessary equipment, and withstand the conditions to which they will be exposed—all of this within a controlled monetary budget.

Ask If a cannonball is fired horizontally from Newton's mountain at a tangential velocity less than 8 km/s, it soon strikes the ground below. Will its speed of impact be greater than, the same as, or less than its barrel speed? *Greater, because a component of its velocity is along the gravitational field of Earth. Any object tossed horizontally that moves downward will pick up speed.* (State that only if it is fired at 8 km/s will its speed remain at 8 km/s.) **If it is fired at 9 or 10 km/s, how does its speed change?** *Speed decreases because a component of its velocity is against the gravitational field—it is going against gravity.*

Teaching Tip State that for close Earth orbit, tangential satellite speeds must range between 8 km/s and 11.2 km/s.

CONCEPT CHECK A satellite in orbit around Earth traces an oval-shaped path called an ellipse.

Teaching Resources
- **Reading and Study Workbook**
- **Laboratory Manual** *44, 45, 46*
- **Presentation*EXPRESS***
- **Interactive Textbook**

14.4 Energy Conservation and Satellite Motion

Key Terms
apogee, perigee

▶ **Teaching Tip** Sketch a large ellipse on the board to represent an elliptical orbit around Earth. Place Earth in the appropriate place (closer to the perigee than most people would place it—see Figure 14.12). Place a satellite at the perigee, and write a large "KE" beside it. That's where the satellite is traveling fastest. But it's also closest to Earth, so write a small "PE" next to the large "KE." Draw the satellite at other points. Ask for relative values of KE and PE at these points. Express these with the exaggerated-symbol technique. After discussion, erase the sketch.

CONCEPT CHECK The sum of the KE and PE of a satellite is constant at all points along an orbit.

Teaching Resources

- Reading Study Workbook
- Concept-Development Practice Book *14-1*
- Transparencies *24, 25*
- Next-Time Question *14-2*
- Presentation*EXPRESS*
- Interactive Textbook

14.5 Kepler's Laws of Planetary Motion

Key Term
Kepler's laws of planetary motion

think!

The orbital path of a satellite is shown below. In which of the positions *A* through *D* does the satellite have the most KE? Most PE? Most total energy? *Answer: 14.4*

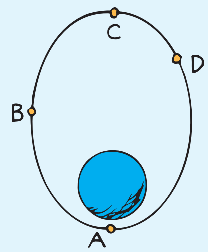
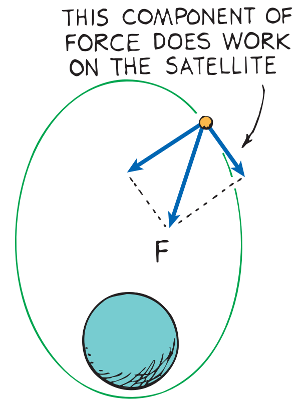


FIGURE 14.13 ▶
In an elliptical orbit, a component of force exists along the direction of the satellite's motion. This component changes the speed and, thus, the KE.



At all points on the orbit—except at the apogee and perigee—there is a component of gravitational force parallel to the direction of satellite motion, as Figure 14.13 shows. This component changes the speed of the satellite. Or we can say: (this component of force) × (distance moved) = change in KE. Either way we look at it, when the satellite gains altitude and moves against this component, its speed and KE decrease. The decrease continues to the apogee. Once past the apogee, the satellite moves in the same direction as the component, and the speed and KE increase. The increase continues until the satellite whips past the perigee and repeats the cycle.

CONCEPT CHECK **What is the relationship between the KE and PE of a satellite in motion?**

14.5 Kepler's Laws Of Planetary Motion

Newton's law of gravitation was preceded by Kepler's laws of planetary motion. **Kepler's laws of planetary motion** are three important discoveries about planetary motion that were made by the German astronomer Johannes Kepler in the beginning of the 1600s. Kepler's career as an astronomer began with a junior assistantship with the famed Danish astronomer Tycho Brahe. Brahe headed the world's first great observatory in Denmark, just prior to the advent of the telescope. Using huge brass protractor-like instruments called *quadrants*, Brahe measured the positions of planets over twenty years so accurately that his measurements are still valid today. Brahe entrusted his data to Kepler. After Brahe's death, Kepler devoted many years of his life to the analysis of Brahe's measurements.

a. Tycho Brahe

b. Johannes Kepler

FIGURE 14.14
a. Tycho Brahe (1546–1601) measured the positions of planets over 20 years so accurately that his measurements are still valid today.
b. Johannes Kepler (1571–1630) devoted many years of his life to the analysis of Brahe's measurements.

Kepler's First Law Kepler's expectation that the planets would move in perfect circles around the sun was shattered after years of effort. He found the paths to be ellipses. **Kepler's first law states that the path of each planet around the sun is an ellipse with the sun at one focus.**

Kepler's Second Law Kepler also found that the planets do not go around the sun at a uniform speed but move faster when they are nearer the sun and more slowly when they are farther from the sun. They accomplish this in such a way that an imaginary line or spoke joining the sun and the planet sweeps out equal areas of space in equal intervals of time. The triangular-shaped areas swept out during a month when a planet is orbiting far from the sun and when a planet is orbiting closer to the sun are shown in Figure 14.15. These two areas are equal. **Kepler's second law states that each planet moves so that an imaginary line drawn from the sun to any planet sweeps out equal areas of space in equal intervals of time.**

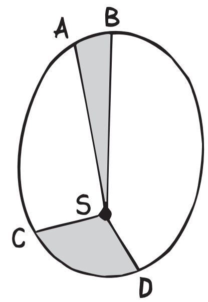
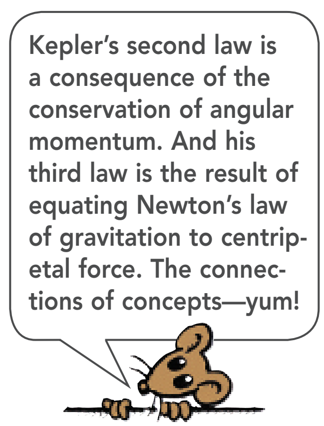


FIGURE 14.15 Equal areas of space are swept out in equal intervals of time.

Kepler was the first to coin the word *satellite*. He had no clear idea *why* the planets moved as he discovered. He lacked a conceptual model. Kepler didn't see that a satellite is simply a projectile under the influence of a gravitational force directed toward the body around which the satellite orbits. You know that if you toss a rock upward, it goes slower the higher it rises because it's going against Earth gravity. And you know that when it returns it's going with gravity and its speed increases. Kepler never realized that a satellite behaves in the same way. Going away from the sun, it slows down. Returning toward the sun, it speeds up. A satellite, whether a planet orbiting the sun, or one of today's satellites orbiting Earth, is slowed going against the gravitational field and sped up going with the field. Kepler wasn't aware of this simplicity, and instead fabricated complex systems of geometrical figures to find sense in his discoveries. These proved futile.



Teaching Tip Kepler's years of data analysis led to his three laws of planetary motion—which illustrates the process of *induction*. Later, Newton came up with his three laws of motion, which in minutes predict all three of Kepler's laws—illustrating the process of *deduction*.

Teaching Tip Kepler's third law is derived from equating centripetal force with gravitational force.

$$\frac{mv^2}{r} = \frac{m(2\pi r)^2}{rT^2}$$

$$\frac{4m\pi^2 r}{T^2} = \frac{GmM}{r^2}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$GM/4\pi^2$ is a constant. Because the ratio r^3/T^2 is constant, the ratio is the same for any distances from mass M. This, in fact, is how the mass of a planet M is made. Simply measure r^3/T^2 and calculate mass M.

Teaching Tip What took Kepler 11 years to discover is calculated with Newton's laws in minutes! (What are we searching for today that tomorrow will be found in the same short time?) In 1611, Kepler wrote a paper on six-cornered snowflakes. Fifty-four years later, Robert Hooke used his early microscope to sketch the forms of snowflakes.

CONCEPT CHECK Kepler's first law states that the path of each planet around the sun is an ellipse with the sun at one focus. Kepler's second law states that each planet moves so that an imaginary line drawn from the sun to any planet sweeps out equal areas of space in equal time intervals. Kepler's third law states that the square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet from the sun.

Teaching Resources

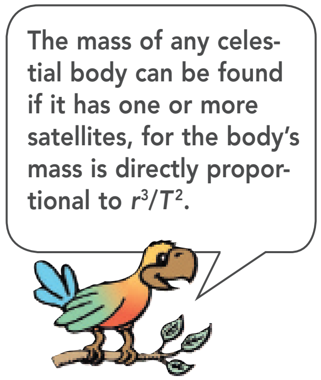
- **Reading and Study Workbook**
- **Presentation*EXPRESS***
- **Interactive Textbook**

14.6 Escape Speed

Key Term

escape speed

▶ **Teaching Tip** Toss something straight upward. Point out that if air resistance does not play a role, then the launching speed and the speed of return are the same. A projectile fired upward at 8 km/s will result in a return speed of 8 km/s. KE lost going up is equal to the KE gained in returning. Beyond 11.2 km/s, the speed is sufficient for a no-return situation. This is escape speed.



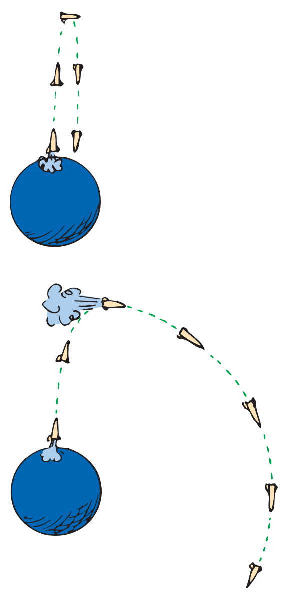


FIGURE 14.16 ▲
The initial thrust of the rocket lifts it vertically. Another thrust tips it from its vertical course. When it is moving horizontally, it is boosted to the required speed for orbit.

Kepler's Third Law After ten years of searching by trial and error for a connection between the time it takes a planet to orbit the sun and its distance from the sun, Kepler discovered a third law. From Brahe's data, Kepler found that the square of any planet's period (T) is directly proportional to the cube of its average orbital radius (r). **Kepler's third law states that the square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet from the sun.** This means that the ratio T^2/r^3 is the same for all planets. So if a planet's period is known, its average orbital radial distance is easily calculated (or vice versa). Kepler's laws apply not only to planets but also to moons or any satellite in orbit around any body. The elliptical orbits of the planets are very nearly circular. Only the precise measurements of Brahe showed the slight differences.

It is interesting to note that Kepler was familiar with Galileo's concepts of inertia and accelerated motion, but he failed to apply them to his own work. Like Aristotle, he thought that the force on a moving body would be in the same direction as the body's motion. Kepler never appreciated the concept of inertia. Galileo, on the other hand, never appreciated Kepler's work and held to his conviction that the planets move in circles.[14.6] Further understanding of planetary motion required someone who could integrate the findings of these two great scientists. The rest is history, for as we have seen, this task was later taken up by Isaac Newton.

CONCEPT CHECK What are Kepler's three laws of planetary motion?

14.6 Escape Speed

When a payload is put into Earth-orbit by a rocket, the speed and direction of the rocket are very important. For example, what would happen if the rocket were launched vertically and quickly achieved a speed of 8 km/s? Everyone had better get out of the way, because it would soon come crashing back at 8 km/s. As Figure 14.16 shows, to achieve orbit, the payload must be launched *horizontally* at 8 km/s once above air resistance. Launched vertically, the old saying "What goes up must come down" becomes a sad fact of life.

Earth But isn't there some vertical speed that is sufficient to ensure that what goes up will escape and not come down? The answer is yes. Neglecting air resistance, fire anything at any speed greater than 11.2 km/s, and it will leave Earth, going more and more slowly, but never stopping.[14.6.1] Let's look at this from an energy point of view.

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How much work is required to move a payload against the force of Earth's gravity to a distance very, very far ("infinitely far") away? The PE is not infinite because the distance is infinite. But gravity diminishes rapidly with distance via the inverse-square law. Most of the work done in launching a rocket, for example, occurs near Earth. It turns out that the value of PE for a 1-kilogram mass infinitely far away is 62 million joules (MJ). So to put a payload infinitely far from Earth's surface requires at least 62 MJ of energy per kilogram of load. A KE per unit mass of 62 MJ/kg corresponds to a speed of 11.2 km/s. This is the value of the escape speed from the surface of Earth.[14.6.2] The **escape speed** is the minimum speed necessary for an object to escape permanently from a gravitational field that holds it.

If we give a payload any more energy than 62 MJ/kg at the surface of Earth or, equivalently, any greater speed than 11.2 km/s, then, neglecting air resistance, the payload will escape from Earth never to return. As it continues outward, its PE increases and its KE decreases. Its speed becomes less and less, though it is never reduced to zero. The payload outruns the gravity of Earth. It escapes.

The Solar System The escape speeds of various bodies in the solar system are shown in Table 14.1. Note that the escape speed from the sun is 620 km/s at the surface of the sun. Even at a distance equaling that of Earth's orbit, the escape speed from the sun is 42.2 km/s. The escape speed values in the table ignore the forces exerted by other bodies. A projectile fired from Earth at 11.2 km/s, for example, escapes Earth but not necessarily the moon, and certainly not the sun.

Table 14.1 Escape Speeds at the Surface of Bodies in the Solar System

Astronomical Body	Mass (Earth masses)	Radius (Earth radii)	Escape Speed (km/s)
Sun	333,000	109	620
Sun (at a distance of Earth's orbit)	333,000	23,500	42.2
Jupiter	318	11	60.2
Saturn	95.2	9.2	36.0
Neptune	17.3	3.47	24.9
Uranus	14.5	3.7	22.3
Earth	1.00	1.00	11.2
Venus	0.82	0.95	10.4
Mars	0.11	0.53	5.0
Mercury	0.055	0.38	4.3
Moon	0.0123	0.28	2.4

Ask **If an object located at a distance beyond Pluto were dropped from a position of rest to Earth, what would be its maximum speed of impact if its increase in speed were due only to Earth's gravity?** *11.2 km/s, the same speed it would need to bounce from Earth to return to its original distance*

Teaching Tip Acknowledge that the term *escape speed* refers to "ballistic speed," the speed a body must have after the thrusting force ends. If the thrusting force were somehow continuous, then any speed could provide escape if maintained over a sufficient time.

▶ **Teaching Tip** Grains of sand and other meteorites that graze Earth's atmosphere burn up and appear as "falling stars." That's why space vehicles are launched to altitudes above the atmosphere.

Question to ponder: In sending probes to possible civilizations "out there," should we share with them our religious truths? Why or why not? And if so, from which faith?

PAUL

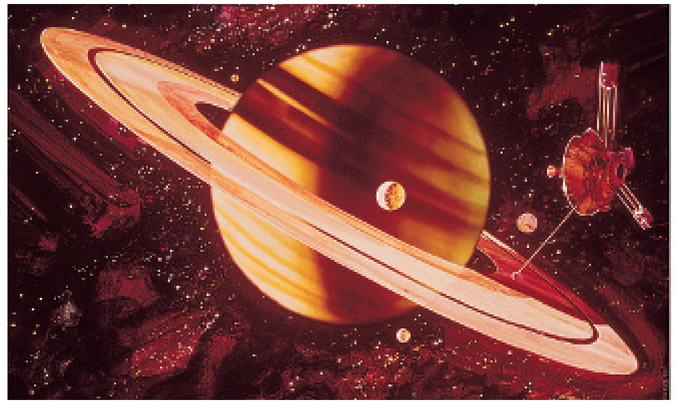

FIGURE 14.17 ▶
Pioneer 10, launched from Earth in 1972, escaped from the solar system in 1984 and is wandering in interstellar space.

The first probe to escape the solar system, *Pioneer 10,* shown in Figure 14.17, was launched from Earth in 1972 with a speed of only 15 km/s. The escape was accomplished by directing the probe into the path of oncoming Jupiter. It was whipped about by Jupiter's great gravitational field, picking up speed in the process—just as the speed of a ball encountering an oncoming bat is increased when it departs from the bat. Its speed of departure from Jupiter was increased enough to exceed the sun's escape speed at the distance of Jupiter. *Pioneer 10* passed the orbit of Pluto in 1984. Unless it collides with another body, it will continue indefinitely through interstellar space. Like a note in a bottle cast into the sea, *Pioneer 10* contains information about Earth that might be of interest to extraterrestrials, in hopes that it will one day wash up and be found on some distant "seashore."

It is important to point out that the escape speeds for different bodies refer to the initial speed given by a brief thrust, after which there is no force to assist motion. But we could escape Earth at any *sustained* speed greater than zero, given enough time. Suppose a rocket is going to a destination such as the moon. If the rocket engines burn out while still close to Earth, the rocket will need a minimum speed of 11.2 km/s. But if the rocket engines can be sustained for long periods of time, the rocket could go to the moon without ever attaining 11.2 km/s.

It is interesting to note that the accuracy with which an unpiloted rocket reaches its destination is accomplished not by staying on a preplanned path, or by getting back on that path if it strays off course. No attempt is made to return the rocket to its planned path. Instead, by communication with the control center, the rocket in effect asks, "Where am I now, and where do I want to go? What is the best way to get there from here, given my present situation?" With the aid of high-speed computers, the answers to these questions are used to find a *new* path. Corrective thrusters put the rocket on this new path. This process is repeated continuously along the way until the rocket reaches its destination.

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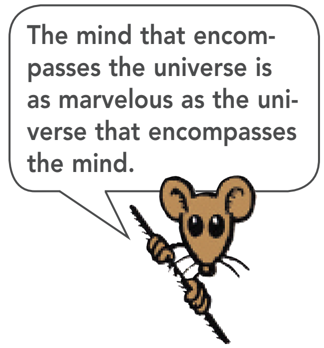
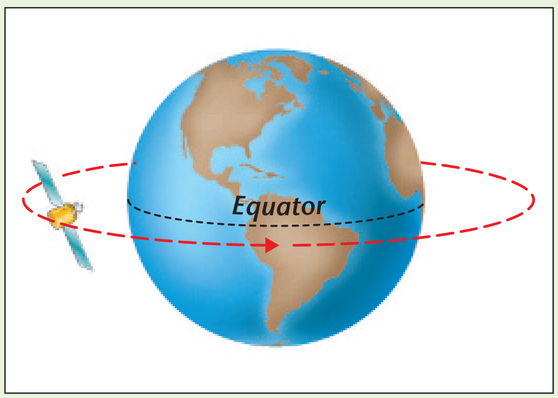
Is there a lesson to be learned here? Suppose you find in your personal life that you are "off course." You may, like the rocket, find it better to take a newer course that leads to your goal as best plotted from your present position and circumstances, rather than try to get back on the course you plotted from a previous position and in, perhaps, different circumstances. So many ideas in physics, it seems, have a moral.



CONCEPT CHECK What condition is necessary for a payload to escape Earth's gravity?

Science, Technology, and Society

Communications Satellites The electromagnetic signals that are broadcast into space to carry television programs or telephone conversations travel in straight lines. In times past these straight-line (often called line-of-sight) communications required tall receiving antenna towers and signal-boosting relay stations on high buildings or mountains. Today many television and telephone signals bounce to us from satellites. These communications satellites are in equatorial orbits with 24-hour periods. Because they revolve once each time Earth rotates once, they appear stationary when we look up at them. These satellites are said to be in geosynchronous orbits.



Dish-shaped antennas almost anywhere on Earth are on a line of sight from one or more communications satellites. Because communications satellites are in equatorial orbit, dish antennas on the equator may tilt east or west, but they don't tilt north or south. An equatorial dish right under a communications satellite is looking straight up. If it held water, it would resemble a birdbath filled to the brim.

Antennas north of the equator must be tilted to the south (and perhaps east or west, too). Those south of the equator must be tilted to the north, and likely east or west as well. Unless you live on the equator, all of the antennas that you see look like partly emptied bowls. In Antarctica or near the North Pole, a dish antenna is tipped so far over that it could hold no water at all.

The fact that geosynchronous satellites remain in one place overhead means it is possible for them to drop vertical cables to Earth's surface where they could be attached. Cables composed of very strong and lightweight carbon-based materials are currently being researched. Rather than being rocketed to the satellite, supplies could be lifted in elevator fashion. Watch for such space elevators in the future!

Critical Thinking Explain why it is not possible for a single communications satellite to serve all parts of Earth.

▶ **Teaching Tip** Discuss why it is advantageous to launch a rocket near the equator, eastward, to take advantage of Earth's spin. State that launches in the United States are most favorable in the southernmost part of Hawaii.

▶ **Teaching Tidbit** Thirty GPS satellites orbit about 20,000 km above Earth. Information on position is accurate to within 10 to 20 cm anywhere in the world.

CONCEPT CHECK If we give a payload any more energy than 62 MJ/kg at the surface of Earth or, equivalently, any greater speed than 11.2 km/s, then, neglecting air resistance, the payload will escape from Earth never to return.

Science, Technology, and Society

CRITICAL THINKING A single satellite will not have a line of sight to all parts of Earth.

Teaching Resources

- Reading and Study Workbook
- Problem-Solving Exercises in Physics *8-3*
- Presentation*EXPRESS*
- Interactive Textbook

14 REVIEW

Teaching Resources

- Teacher*EXPRESS*
- Virtual Physics Lab *17*
- Conceptual Physics Alive! DVDs *Satellite Motion*

14 REVIEW

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Concept Summary

- A stone thrown fast enough to go a horizontal distance of 8000 meters during the time (1 second) it takes to fall 5 meters will orbit Earth.
- A satellite in circular orbit around Earth is always moving perpendicular to gravity and parallel to Earth's surface at constant speed.
- A satellite in orbit around Earth traces and oval-shaped path called an ellipse.
- The sum of the KE and PE of a satellite is constant at all points along an orbit.
- Kepler's first law states that the path of each planet around the sun is an ellipse with the sun at one focus.
- Kepler's second law states that each planet moves so that an imaginary line drawn from the sun to any planet sweeps out equal areas of space in equal intervals of time.
- Kepler's third law states that the square of the orbital period of a planet is directly proportional to the cube of the average distance of the planet from the sun. ($T^2 \sim r^3$ for all planets)
- If we give a payload any more energy than 62 MJ/kg at the surface of Earth or, equivalently, any greater speed than 11.2 km/s, then, neglecting air resistance, the payload will escape from Earth never to return.

Key Terms

satellite *(p. 263)*
period *(p. 266)*
ellipse *(p. 267)*
focus (pl. **foci**) *(p. 267)*
apogee *(p. 269)*
perigee *(p. 269)*
Kepler's laws of planetary motion *(p. 270)*
escape speed *(p. 273)*

think! Answers

14.2 In each second, the satellite falls about 5 m below the straight-line tangent it would have taken if there were no gravity. Earth's surface curves 5 m below an 8-km straight-line tangent. Since the satellite moves at 8 km/s, it "falls" at the same rate Earth "curves."

14.3 The satellite has its greatest speed as it whips around *A*. It has its least speed at *C*. Beyond *C*, it gains speed as it falls back to *A* to repeat its cycle.

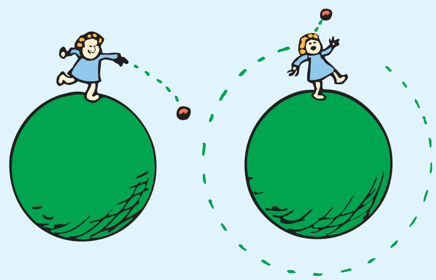
14.4 The KE is maximum at *A;* the PE is maximum at *C;* the total energy is the same anywhere in the orbit.

14 ASSESS

Check Concepts

Section 14.1

1. If we drop a ball from rest, how far will it fall vertically in the first second? If we instead move our hand horizontally and drop it (throw it), how far will it fall vertically in the first second?



2. What do the distances 8000 m and 5 m have to do with a line tangent to Earth's surface?

Section 14.2

3. How does the direction of motion of a satellite in circular orbit compare with the curve of Earth's surface?

4. Why doesn't gravitational force change the speed of a satellite in circular orbit?

5. Does the period of a satellite in a circular orbit increase or decrease as its distance from Earth increases?

Section 14.3

6. Describe an ellipse.

Section 14.4

7. Why does gravitational force change the speed of a satellite in elliptical orbit?

8. **a.** Where in an elliptical orbit is the speed of a satellite maximum?
   b. Where is it minimum?

9. The sum of PE and KE for a satellite in a circular orbit is constant. Is this sum also constant for a satellite in an elliptical orbit?

10. Why does the force of gravity do no work on a satellite in circular orbit, but does do work on a satellite in an elliptical orbit?

Section 14.5

11. What scientist gathered accurate data on planetary paths around the sun? What scientist discovered that the paths are ellipses? What scientist explained the ellipses?

12. When is the speed of a satellite greatest, when closer to Earth or farther from Earth?

13. What is the mathematical relationship between how long it takes a planet to orbit the sun and its distance from the sun?

14 ASSESS

Check Concepts

1. 5 m in both cases
2. Earth's surface drops 5 m below a tangent line in 8000 m.
3. Path is parallel to the surface below.
4. There is no component of force in the direction of motion.
5. increase
6. The sum of the distances from each point on a closed curve to the two foci is constant.
7. There is a component of force in the direction of motion.
8. **a.** perigee (closest to Earth)
   b. apogee (farthest from Earth)
9. yes
10. Circular orbit—there is no force component in the direction of motion; elliptical orbit—there is a force component in the direction of motion.
11. Brahe, Kepler, Newton
12. Faster when close
13. $T^2 \sim r^3$

14. **a.** 8 km/s
b. 11.2 km/s
c. When an object exceeds the maximum speed, it will escape.

15. It will return; it will escape.

16. **a.** 620 km/s
b. 42.2 km/s

17. 2.4 km/s

18. Yes; if the upward force continued

19. By directing the probe into the path of oncoming Jupiter

Think and Rank

20. **a.** A, B, C
b. C, B, A

21. **a.** A, B, C, D
b. D, C, B, A
c. No

14 ASSESS *(continued)*

Section 14.6

14. **a.** What is the minimum speed for circling Earth in close orbit?
b. What is the maximum speed in an orbit that comes close to Earth at one point?
c. What happens above the maximum speed?

15. Neglecting air resistance, what will happen to a projectile that is fired vertically at 8 km/s? At 12 km/s?

16. **a.** How fast would a particle have to be ejected from the sun to leave the solar system?
b. What speed would be needed if an ejected particle started at a distance from the sun equal to Earth's distance from the sun?

17. What is the escape speed on the moon?

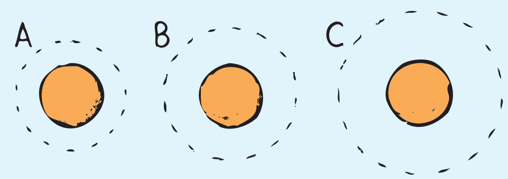
18. Although the escape speed from the surface of Earth is 11.2 km/s, couldn't a rocket with enough fuel escape at any speed? Why or why not?

19. How was *Pioneer 10* able to escape the solar system with an initial speed less than escape speed?

Think and Rank

Rank each of the following sets of scenarios in order of the quantity or property involved. List them from left to right. If scenarios have equal rankings, then separate them with an equal sign. (e.g., A = B)

20. The dashed lines show three circular orbits about Earth.



Rank the following quantities for these orbits from greatest to least.
a. orbital speed
b. time for orbiting Earth

21. Four satellites in circular orbit about Earth have the following characteristics:

(A) m = 4000 kg; height 300 km
(B) m = 5000 kg; height 350 km
(C) m = 4000 kg; height 400 km
(D) m = 5000 kg; height 500 km

a. Rank the satellites' orbital speeds from greatest to least.
b. Rank the satellites' times for orbiting Earth from greatest to least.
c. Does mass affect your answers to parts (a) and (b)?

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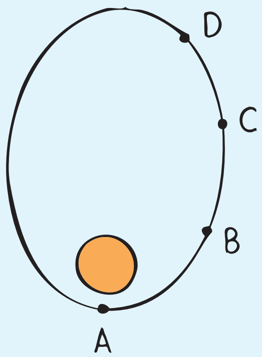
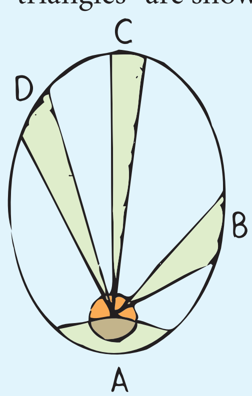
22. The positions of a satellite in elliptical orbit are indicated.



Rank these quantities from greatest to least.

a. gravitational force
b. speed
c. momentum
d. KE
e. PE
f. total energy (KE + PE)
g. acceleration

23. Kepler tells us that a planet sweeps out equal areas in equal intervals of time. Four such equal-area "triangles" are shown.



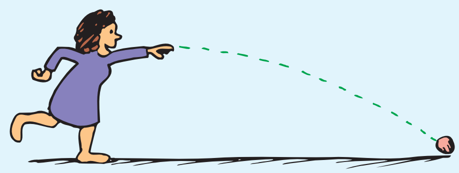
Rank these quantities from greatest to least.

a. average speed during the time interval
b. acceleration during the time interval

Think and Explain

24. A satellite can orbit at 5 km above the moon, but not at 5 km above Earth. Why?

25. Does the speed of a satellite around Earth depend on its mass? Its distance from Earth? The mass of Earth?



26. If a cannonball is fired from a tall mountain, gravity changes its speed all along its trajectory. But if it is fired fast enough to go into circular orbit, gravity does not change its speed at all. Why?

27. Does gravity do any *net* work on a satellite in an elliptical orbit during one full orbit? Explain your answer.

28. A geosynchronous Earth satellite can remain almost directly overhead in Singapore, but not San Francisco, Chicago, or New York City. Why?

29. If you stopped an Earth satellite dead in its tracks, it would simply crash into Earth. Why, then, don't the communications satellites that hover motionless above the same spot on Earth crash into Earth?

22. a. A, B, C, D
b. A, B, C, D
c. A, B, C, D
d. A, B, C, D
e. D, C, B, A
f. A = B = C = D
g. A, B, C, D

23. a. A, B, D, C
b. A, B, D, C

Think and Explain

24. Too much air resistance

25. No; yes; yes; $v = \sqrt{GM/d}$

26. In circular orbit there is no force component in its direction of motion to speed it up.

27. No; after one complete orbit, its KE returns to the same value it had before, and no net work is done.

28. A geosynchronous satellite orbits above the equator. Singapore is on equator, but the other cities are not.

29. They are moving in their orbits and just appear motionless.

30. Half brought to rest falls vertically to Earth. Other half has twice former speed (conservation of momentum).

31. Less; the gravitational field near the moon's surface is weaker than the gravitational field near Earth's surface.

32. The greater tangential Earth velocity assists the launch.

33. It uses Earth's spin to boost its speed.

34. Mercury shorter; Uranus longer

35. Same as escape speed: 11.2 km/s

36. The ship with its first stage fully loaded with fuel still intact is more massive. Also, g is larger.

37. At the same speed it would take to send it from the sun to its present location; this is only a little less than escape speed from the sun, 620 km/s, since Pluto is so far from the sun.

38. Launch the object backward at orbital speed. Then, with respect to Earth, its tangential speed would be zero and it would then drop to Earth below.

39. $v = \sqrt{2GM/d}$, so greater mass means that the escape speed is greater.

Think and Solve

40. $d = 2\pi r$ and 1 year = 31,536,000 s, so $v = 2\pi r/T = 2\pi(1.5 \times 10^{11}\text{ m})/(31{,}536{,}000\text{ s}) = 3 \times 10^4$ m/s.

41. **a.** Refer to Note 14.2. Substitute $v = 2\pi r/T$ into $mv^2/r = mMG/r^2$ and solve for T, so that $T = 2\pi\sqrt{r^3/GM} = 8000\text{ s} = 2.2\text{ h}$.
b. $v = 2\pi r/T = 2\pi(2.0 \times 10^6\text{ m/s})/8000\text{s} = 1600\text{ m/s} = 5800\text{ km/h}$

14 ASSESS *(continued)*

30. In an accidental explosion, a satellite breaks in half while in circular orbit about Earth. One half is brought momentarily to rest. What is the fate of the half brought to rest? What is the speed of the other half? (*Hint:* Think momentum conservation.)

31. Would you expect the speed of a satellite in close circular orbit about the moon to be less than, equal to, or greater than 8 km/s? Why?

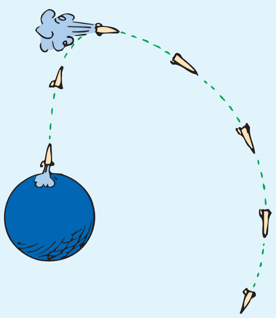
32. Why do you suppose that sites close to the equator are preferred for launching satellites? (*Hint:* Look at the spinning Earth from above either pole and compare it to a spinning turntable.)

33. Why do you suppose that a space shuttle is sent into orbit by firing it in an easterly direction (the direction in which Earth spins)?

34. Consider two planets: Mercury, close to the sun, and Uranus, far from the sun. Which of these planets has a period shorter than Earth's period around the sun? Which has a period longer than Earth's?

35. What is the maximum possible speed of impact upon Earth's surface for a faraway object initially at rest that falls to Earth due only to Earth's gravity?

36. Why does most of the work done in launching a rocket take place when the rocket is still close to Earth's surface?



37. If Pluto were somehow stopped short in its orbit, it would fall into the sun rather than around it. About how fast would it be moving when it hit the sun?

38. If an astronaut in an orbiting space shuttle wished to drop something to Earth, how could this be accomplished?

39. If Earth somehow acquired more mass, with no change in its radius, would escape speed be less than, equal to, or more than 11.2 km/s? Why?

Think and Solve

40. Calculate the speed in m/s at which Earth revolves around the sun. Note: The orbit is nearly circular.

41. A spaceship in circular orbit about the moon is 2.0×10^6 m from its center.
a. Show that the period of the spaceship is 2.2 h.
b. Show that the speed of the spaceship relative to the moon is about 5800 km/h.

42. Calculate the speed in m/s at which the moon revolves around Earth. Note: The orbit is nearly circular.

43. At a particular point, a satellite in an elliptical orbit has a gravitational potential energy of 5000 MJ with respect to Earth's surface and a kinetic energy of 4500 MJ. At another point in its orbit, the satellite's potential energy is 6000 MJ. What is its kinetic energy at that point?

44. An orbiting satellite of mass *m* is pulled toward Earth by a force *ma*. Equate *ma* to the force in Newton's equation for universal gravitation and show that the satellite's acceleration is $a = \frac{GM}{d^2}$.

45. The force of gravity between Earth and an Earth satellite is given by $F = G\frac{mM}{d^2}$, where *m* is the mass of the satellite, *M* is the mass of Earth, and *d* is the distance between the satellite and the center of Earth. If the satellite follows a circular orbit, the force keeping it in orbit must be the centripetal force, given by $F = \frac{mv^2}{r}$. Equate the two expressions for force to show that the speed is $v = \sqrt{\frac{GM}{d}}$.

46. Use the result of Question 45 (now with the sun instead of Earth as the center of force) to calculate the speed in m/s at which Earth revolves about the sun. Assume Earth's orbit is nearly circular.

47. In 1610, Galileo discovered four moons of Jupiter. (Today we know that there are more than 60!) Io, the innermost of the moons observed by Galileo, is 4.2×10^8 m from Jupiter's center and has a period of 1.5×10^5 s. Calculate Jupiter's mass.

48. A planet in a circular orbit takes a time *T* to orbit its sun at a radial distance *r*. In terms of *r* and *T*, how fast is the planet moving in its orbit?

49. The speed of a satellite in a circular orbit is given by the equation $v = \sqrt{\frac{GM}{r}}$, where *G* is the gravitational constant, *M* is the mass of Earth, and *r* is the radial distance between the satellite and the center of Earth. Equate this to the other expression for speed, $v = \frac{d}{t}$. Find the equation for the time the satellite takes to completely orbit Earth—the period *T*. Use the circumference of the complete orbit, $2\pi r$, for the distance traveled, and *T* for the period of rotation.

50. Use the equation $T = 2\pi\sqrt{\frac{r^3}{GM}}$ to show that the period of the space shuttle 200 km above Earth's surface is about 90 minutes.

More Problem-Solving Practice
Appendix F

42. Here the period is 27.3 days, so $v = 2\pi r/T = 2\pi(3.85 \times 10^8\ \text{m})/(27.3 \times 24 \times 3600)\ \text{s} = 1030$ m/s.

43. By the conservation of energy, total energy is 5000 MJ + 4500 MJ = 9500 MJ. So when PE becomes 6000 MJ, KE is 9500 MJ − 6000 MJ = 3500 MJ.

44. $ma = GmM/d^2$ so $a = GM/d^2$.

45. $GmM/d^2 = mv^2/d$; canceling *m* and *d* we get $GM/d = v^2$ so $v = \sqrt{GM/d}$.

46. $v = \sqrt{GM_{sun}/d_{Earth-sun}} = 3.0 \times 10^4$ m/s = 30 km/s

47. $v = 2\pi r/T = \sqrt{GM/d}$ so $M = 4\pi^2 r^3/GT = 1.9 \times 10^{27}$ kg

48. v = distance/time = $2r\pi/T$

49. $v = \sqrt{GM/r} = 2\pi r/T$, so $\sqrt{r/GM} = T/2\pi r$; solve for *T*: $T = 2\pi r\sqrt{r/GM}$. Since $r = \sqrt{r^2}$, this can be written as $T = 2\pi\sqrt{r^3/GM}$.

50. $r = 6.38 \times 10^6\text{m} + 0.2 \times 10^6\text{m} = 6.58 \times 10^6\text{m}$. $T = 2\pi\sqrt{r^3/GM} = 5300$ s = 88 min.

Teaching Resources

- Computer Test Bank
- Chapter and Unit Tests