Vectors and Scalars

Physics

Scalar

A **SCALAR** is ANY quantity in physics that has MAGNITUDE, but NOT a direction associated with it.

Magnitude – A numerical value with units.

Scalar Example	Magnitude
Speed	20 m/s
Distance	10 m
Age	15 years
Heat	1000 calories

Vector

A **VECTOR** is ANY quantity in physics that has BOTH MAGNITUDE and DIRECTION.

Vector	Magnitude & Direction
Velocity	20 m/s, N
Acceleration	10 m/s/s, E
Force	5 N, West

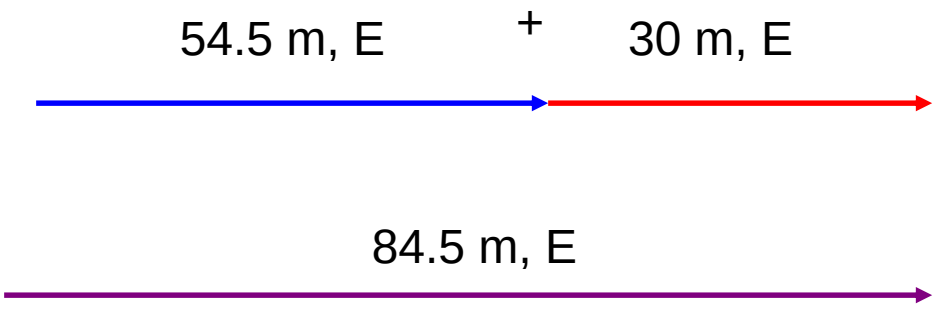
$$\vec{v}, \vec{x}, \vec{a}, \vec{F}$$

Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.

Applications of Vectors

VECTOR ADDITION – If 2 similar vectors point in the SAME direction, add them.

- **Example: A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?**

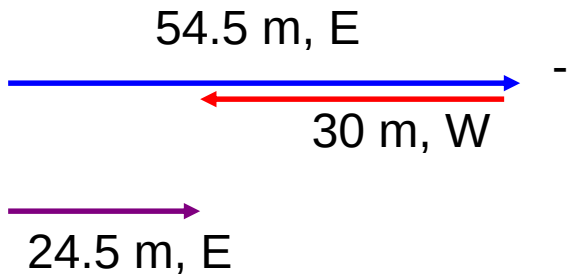


Notice that the SIZE of the arrow conveys MAGNITUDE and the way it was drawn conveys DIRECTION.

Applications of Vectors

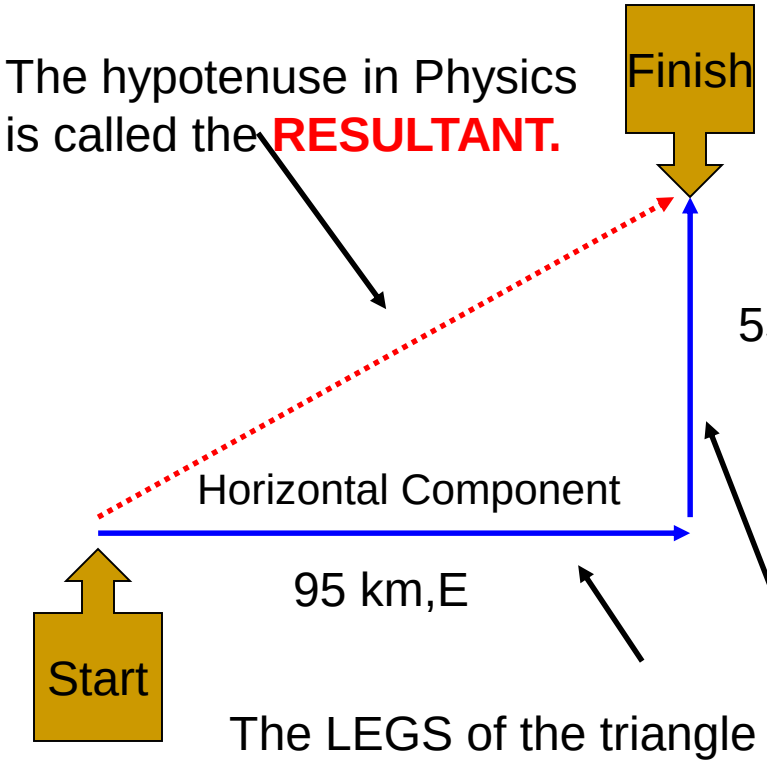
VECTOR SUBTRACTION - If 2 vectors are going in opposite directions, you **SUBTRACT.**

- **Example: A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started?**



Non-Collinear Vectors

When 2 vectors are perpendicular, you must use the Pythagorean theorem.



A man walks 95 km, East then 55 km, north. Calculate his RESULTANT DISPLACEMENT.

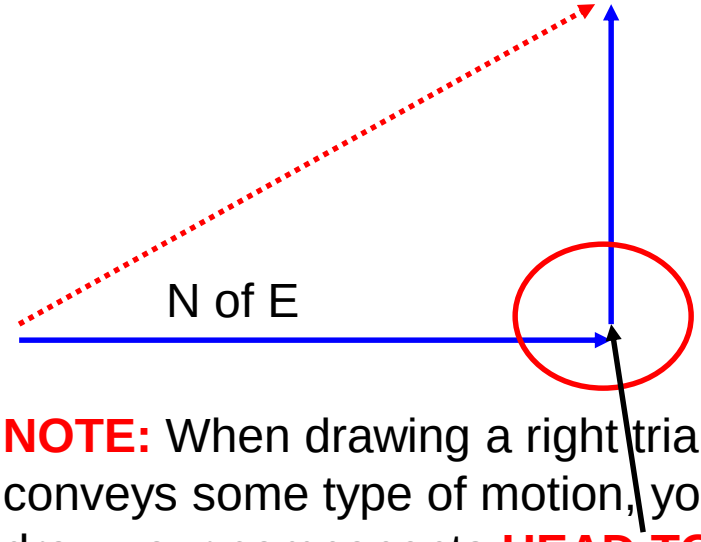
$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$

$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$

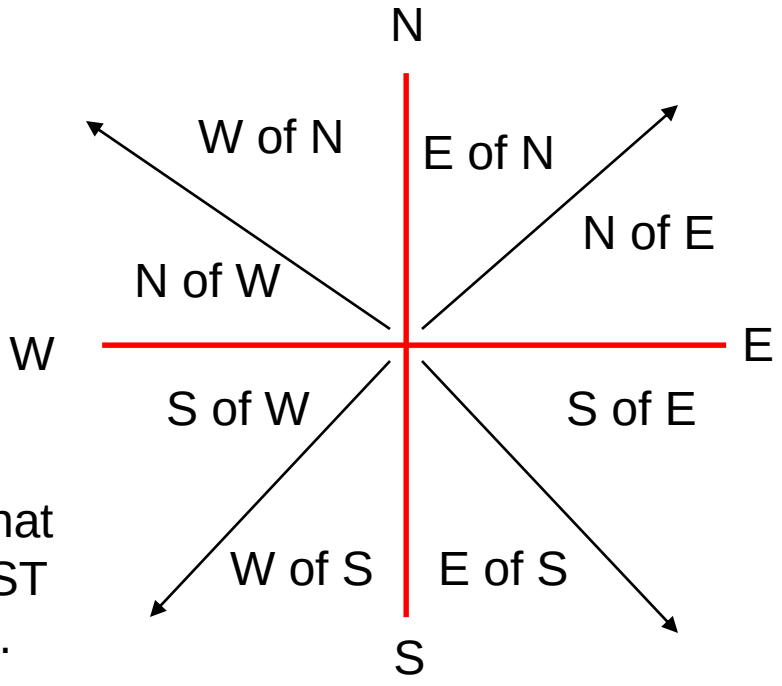
$$c = \sqrt{12050} = 109.8\ km$$

BUT……what about the direction?

In the previous example, DISPLACEMENT was asked for and since it is a VECTOR we should include a DIRECTION on our final answer.

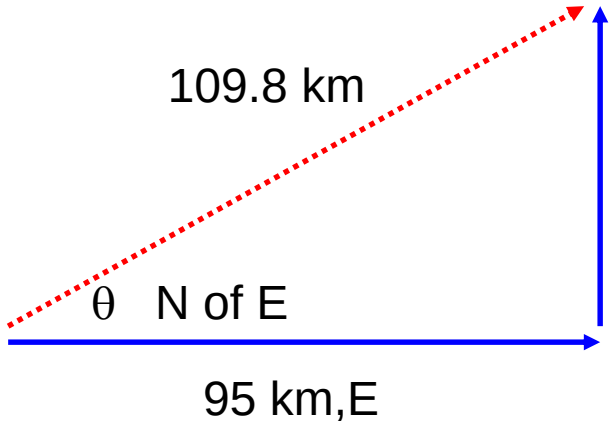


NOTE: When drawing a right triangle that conveys some type of motion, you MUST draw your components **HEAD TO TOE**.



BUT…..what about the VALUE of the angle???

Just putting North of East on the answer is NOT specific enough for the direction. We MUST find the VALUE of the angle.



To find the value of the angle we use a Trig function called TANGENT.

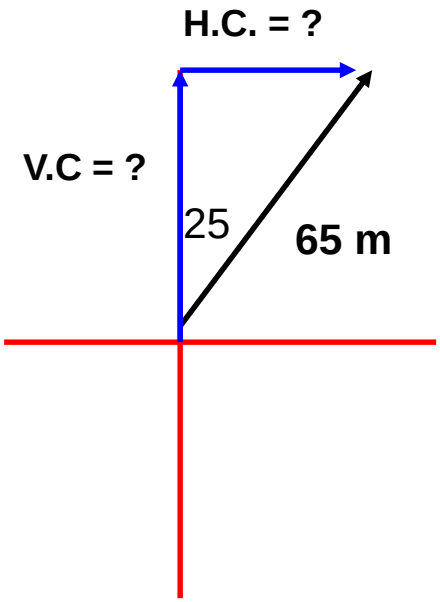
$$Tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{55}{95} = 0.5789$$

$$\theta = Tan^{-1}(0.5789) = 30^{\circ}$$

So the COMPLETE final answer is : **109.8 km, 30 degrees**

What if you are missing a component?

Suppose a person walked 65 m, 25 degrees East of North. What were his horizontal and vertical components?



The goal: **ALWAYS MAKE A RIGHT TRIANGLE!**

To solve for components, we often use the trig functions sine and cosine.

$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

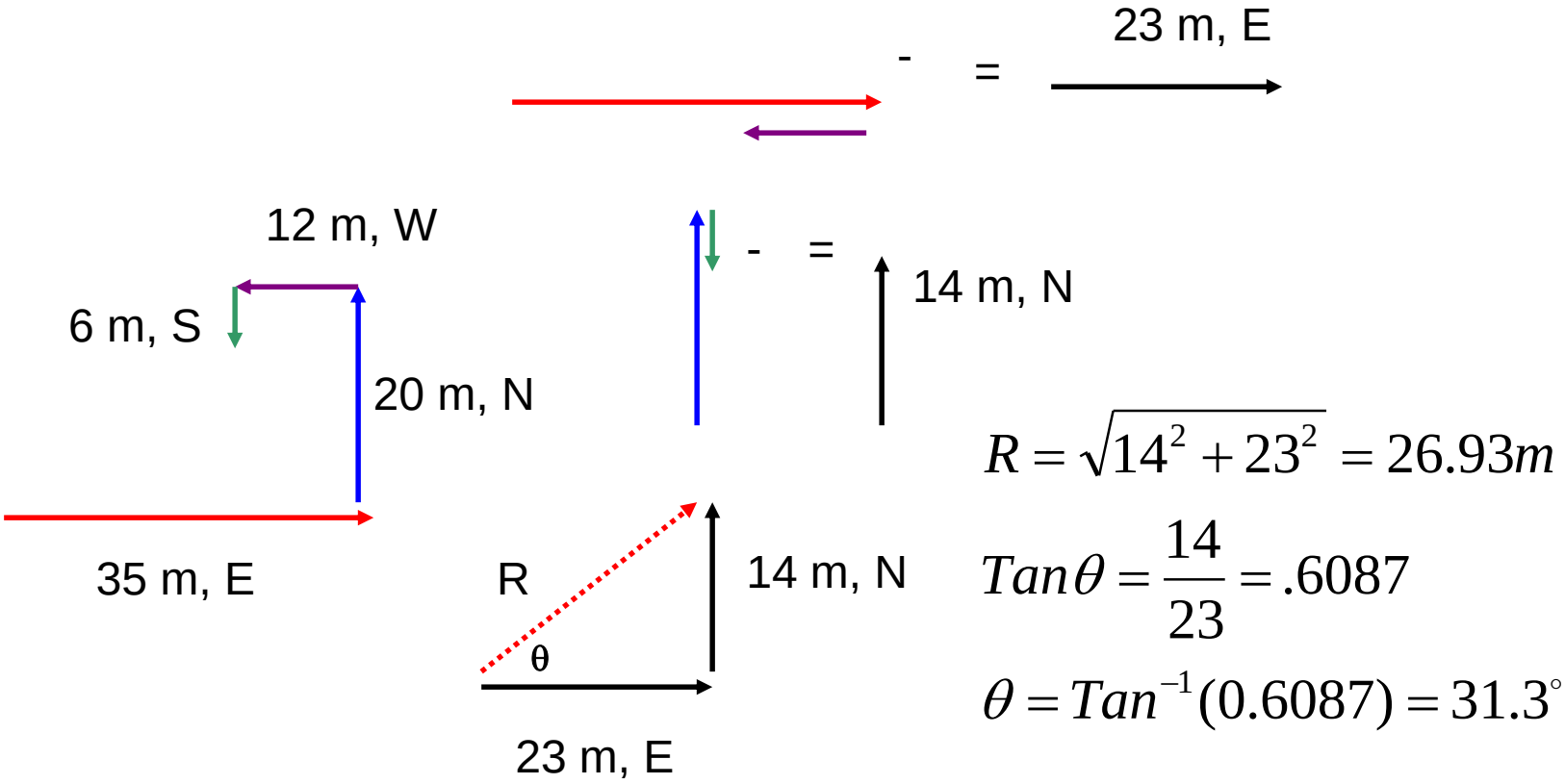
$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = V.C. = 65\cos 25 = 58.91m, N$$

$$opp = H.C. = 65\sin 25 = 27.47m, E$$

Example

A bear, searching for food wanders 35 meters east then 20 meters north. Frustrated, he wanders another 12 meters west then 6 meters south. Calculate the bear's displacement.



$$R = \sqrt{14^2 + 23^2} = 26.93m$$

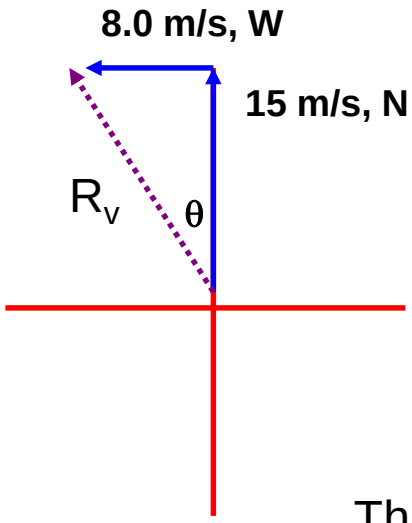
$$Tan\theta = \frac{14}{23} = .6087$$

$$\theta = Tan^{-1}(0.6087) = 31.3^\circ$$

The Final Answer: **26.93 m, 31.3 degrees**

Example

A boat moves with a velocity of 15 m/s, N in a river which flows with a velocity of 8.0 m/s, west. Calculate the boat's resultant velocity with respect to due north.



$$R_v = \sqrt{8^2 + 15^2} = 17\, m/s$$

$$Tan\theta = \frac{8}{15} = 0.5333$$

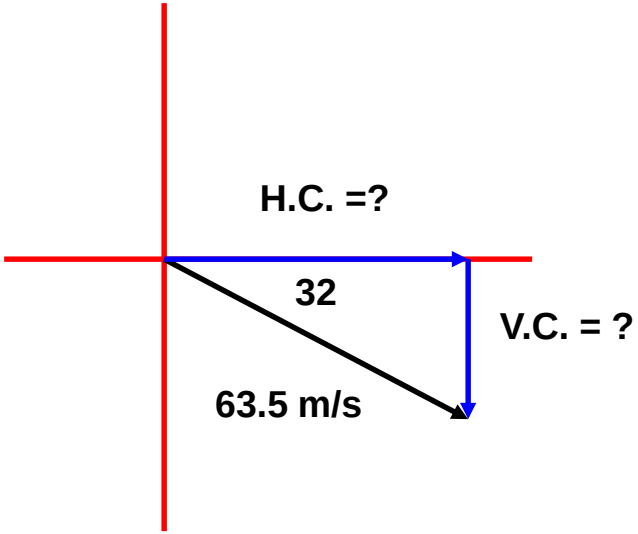
$$\theta = Tan^{-1}(0.5333) = 28.1^\circ$$

$$90^\circ + 28.1^\circ = 118.1^\circ$$

The Final Answer : **17 m/s, @ 118.1 degrees**

Example

A plane moves with a velocity of 63.5 m/s at 32 degrees South of East. Calculate the plane's horizontal and vertical velocity components.



$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

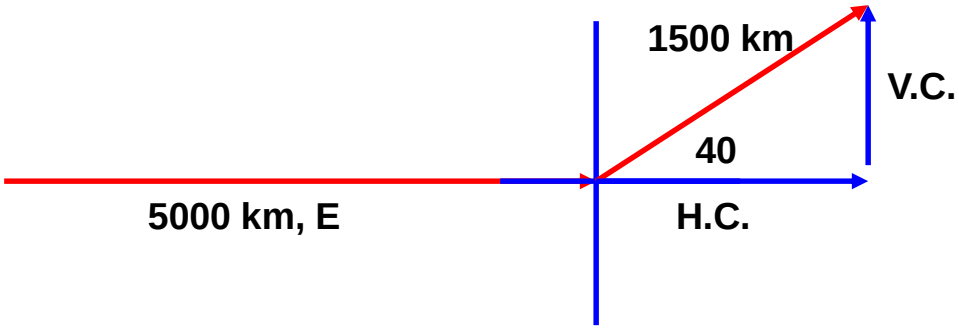
$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 63.5\cos 32 = 53.85\,m/s, E$$

$$opp = V.C. = 63.5\sin 32 = 33.64\,m/s, S$$

Example

A storm system moves 5000 km due east, then shifts course at 40 degrees North of East for 1500 km. Calculate the storm's resultant displacement.

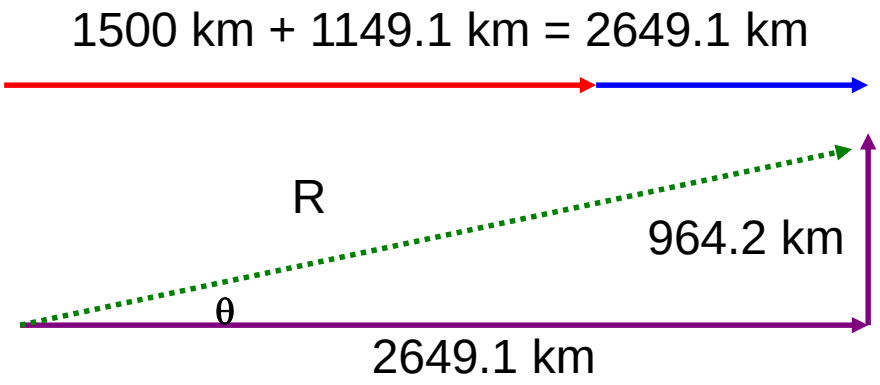


$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$

$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 1500\cos 40 = 1149.1\ km, E$$

$$opp = V.C. = 1500\sin 40 = 964.2\ km, N$$



$$R = \sqrt{2649.1^2 + 964.2^2} = 2819.1 km$$

$$Tan\theta = \frac{964.2}{2649.1} = 0.364$$

$$\theta = Tan^{-1}(0.364) = 20.0^\circ$$

The Final Answer: **2819.1 km @ 20 degrees, East of North**