

Work each of the following problems. SHOW ALL WORK.

Teacher Notes: Questions 1-7 deal with one- and two-dimensional vector addition, including the Pythagorean theorem. Questions 8-10 require the use of right-triangle trigonometry in order to solve.

1. A cart rolls 2 m to the right then rolls back 1 m to the left.

- a. What is the total distance rolled by the cart?

The total distance is 3 m.

- b. What is the displacement of the cart from the initial to the final positions?

The displacement is 1 m to the right.

2. A child observes a caterpillar walking on a window sill. The caterpillar walks 18 cm to the left, then 6 cm to the right, then 10 cm to the left.

- a. What is the total distance walked by the caterpillar?

The total distance is 34 cm.

- b. What is the displacement of the caterpillar?

The displacement is 22 cm to the left.

3. A ball is thrown upward from an initial height of 1.5 m. The ball reaches a height of 5 m then falls to the ground.

- a. What is the total distance traveled by the ball?

The total distance traveled is 8.5 m: The ball rises 3.5 m to its highest point,

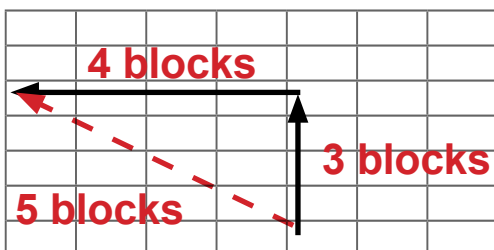
falls 3.5 m back to its original position, then falls an additional 1.5 m to the ground.

- b. What is the displacement of the ball?

If students set the ball's initial position as the origin, and the positive direction as up,

then the displacement of the ball is -1.5 m (1.5 m below the origin).

4. The path from the subway station to the art museum is three blocks to the north then four blocks to the west. What is the straight-line distance in blocks from the subway station to the art museum?



In order to calculate the displacement between the subway station and the museum, students will use the Pythagorean theorem.

$$R^2 = \sqrt{(\sum x)^2 + (\sum y)^2}$$

$$R^2 = \sqrt{(-4 \text{ blocks})^2 + (3 \text{ blocks})^2}$$

$$R^2 = \sqrt{25 \text{ blocks}^2}$$

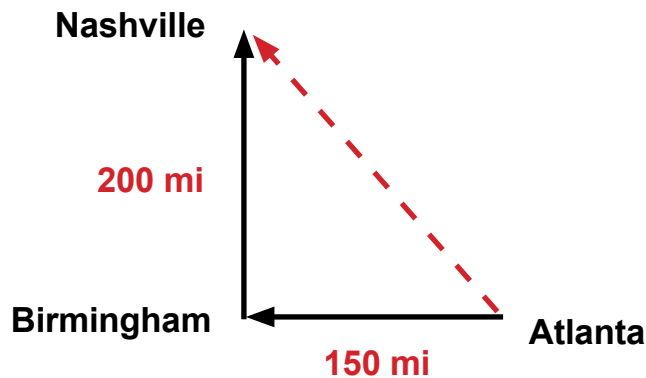
$$R = 5 \text{ blocks}$$

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Unit 2A_Practice Problems TEACHER

Work each of the following problems. SHOW ALL WORK.

5. When looking at a map, a student realizes that Birmingham is nearly due west of Atlanta, and Nashville is nearly due north of Birmingham. If the distance from Atlanta to Birmingham is roughly 150 mi, and the distance from Birmingham to Nashville is roughly 200 mi, what is the estimated distance from Atlanta to Nashville?



In order to calculate the displacement between Atlanta and Nashville, students will use the Pythagorean theorem.

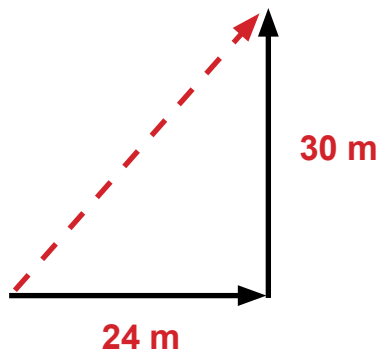
$$R^2 = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$R^2 = \sqrt{(-150 \text{ mi})^2 + (200 \text{ mi})^2}$$

$$R^2 = \sqrt{62\,500 \text{ mi}^2}$$

$$R = 250 \text{ mi}$$

6. A local sign company needs to install a new billboard. The signpost is 30 m tall, and the ladder truck is parked 24 m away from the bottom of the post due to an uneven ravine. How long must the ladder be in order to reach the top of the signpost from the ladder truck?



In order to calculate the displacement between the truck and the signpost, students will use the Pythagorean theorem.

$$R^2 = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$R^2 = \sqrt{(24 \text{ m})^2 + (30 \text{ m})^2}$$

$$R^2 = \sqrt{1476 \text{ m}^2}$$

$$R = 38.4 \text{ m}$$

7. In order to hike around a portion of Lake Allatoona, a tour guide determines that he must take his group 150 m east, 60 m north, then 75 m west. What is the displacement of the tour group from its initial to final position on opposite sides of the lake?

Vector	x-component	y-component
1	150 m	0 m
2	0 m	60 m
3	-75 m	0 m
Total	75 m	60 m

Because more than two directions are involved, students will use a component table to organize their data.

$$R^2 = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$R^2 = \sqrt{(75 \text{ m})^2 + (60 \text{ m})^2}$$

$$R^2 = \sqrt{9\,225 \text{ m}^2}$$

$$R = 96.0 \text{ m}$$

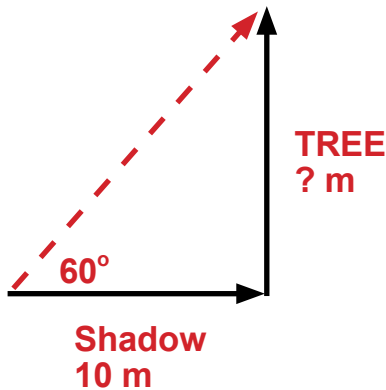
After determining the total x- and y-components, students will use the Pythagorean theorem to solve for the total displacement.

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Unit 2A_Practice Problems TEACHER

Work each of the following problems. **SHOW ALL WORK.**

8. A student wants to estimate the height of a tree from ground level. He measures the length of its shadow as 10 m and the angle from the top of the shadow on the ground to the top of the tree as 60° . What is the height of the tree based on these measurements?



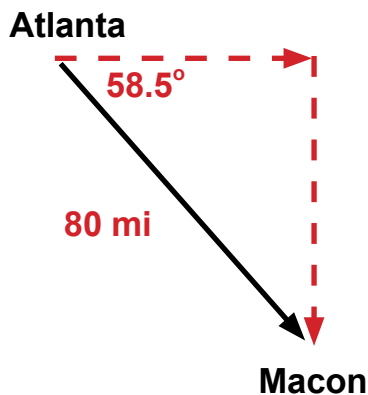
In order to solve for the height of the tree, students will determine which trigonometric function relates the two knowns (the angle and the length of the shadow) and the unknown (the height of the tree).

The trigonometric function that relates these three values is tangent because the length of the shadow is the adjacent side to the angle, and the height of the tree is the opposite side.

$$\tan \Theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{opposite} = (10 \text{ m}) \tan 60^\circ$$

$$\tan 60^\circ = \frac{\text{opposite}}{10 \text{ m}} \quad \text{opposite} = 17.3 \text{ m}$$

9. The distance from Atlanta to Macon is about 80 mi at 58.5° south of east. What distances due east and due south must you drive in order to travel from Atlanta to Macon?



In this question, students will solve for the other two sides of the triangle using two trigonometric identities.

The 80-mile distance is the hypotenuse of the triangle, and the east, or x-component of the vector, is the adjacent side to the angle. So, in order to travel from Atlanta to Macon, you must drive 41.8 mi east and 68.2 mi south.

$$\cos \Theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 58.5^\circ = \frac{\text{adjacent}}{80 \text{ mi}}$$

$$\text{adjacent} = (80 \text{ mi}) \cos 58.5^\circ$$

$$\text{adjacent} = 41.8 \text{ mi}$$

Students will solve for the south, or y-component of the vector, using sine.

$$\sin \Theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 58.5^\circ = \frac{\text{opposite}}{80 \text{ mi}}$$

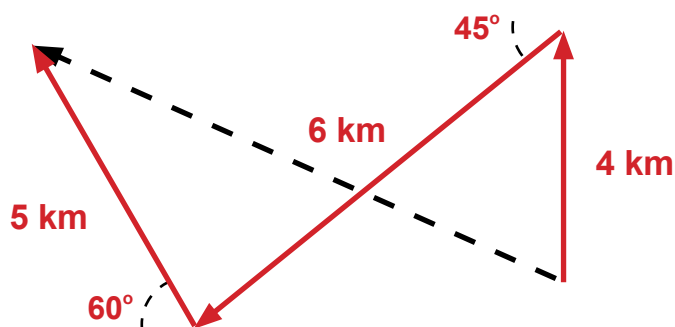
$$\text{opposite} = (80 \text{ mi}) \sin 58.5^\circ$$

$$\text{opposite} = 68.2 \text{ mi}$$

So, in order to travel from Atlanta to Macon, you must drive 41.8 mi east and 68.2 mi south.

Work each of the following problems. SHOW ALL WORK.

10. A pizza delivery driver must make three stops on her route. She will first leave the restaurant and travel 4 km due north to the first house. The next house is 6 km away at 45° south of west according to her map. The final stop is 5 km away at 60° north of west. What is her displacement from the restaurant to the final stop?



Students will use the three known directions and a component table to determine the displacement.

Multiply the total displacement by the cosine of the angle to find the x-components and by the sine of the angle to find the y-components. Students are able to do this because the angle is measured from the horizontal.

Vector	x-component	y-component
1	0 km	4 km
2	6 km (cos 225°) = -4.24 km	6 km (sin 225°) = -4.24 km
3	5 km (cos 120°) = -2.5 km	5 km (sin 120°) = 4.33 km
Total	-6.74 km	4.09 km

After determining the total x- and y-components, students will use the Pythagorean theorem and the inverse tan function to solve for the total displacement.

$$R^2 = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$R^2 = \sqrt{(-6.74 \text{ km})^2 + (4.09 \text{ km})^2}$$

$$R^2 = \sqrt{62.16 \text{ km}^2}$$

$$R = 7.88 \text{ km}$$

$$\Theta = \tan^{-1} \left(\frac{\Sigma y}{\Sigma x} \right)$$

$$\Theta = \tan^{-1} \left(\frac{4.09 \text{ m}}{-6.74 \text{ m}} \right)$$

$$\Theta = \tan^{-1} (-0.61)$$

$$\Theta = -31.25^\circ \text{ Add } 180^\circ \text{ because the resultant lies in the 2nd quadrant.}$$

$$\Theta = -31.25^\circ + 180^\circ$$

$$\Theta = 148.7^\circ \text{ with respect to the positive x-axis}$$